Abstracts

Morning Plenary Address

Inverse problems for differential equations: how to "see" an underground tectonic fault
Dr. Anna Mazzucato, Pennsylvania State University

I will give a brief introduction to inverse problems for differential equations, specifically how to remotely probe an object using solutions to differential equations. Examples of such problems are in medical imaging, non-destructive testing, and mining prospection. In seismology, the interior of the Earth can be probed in different ways, for example by generating small earthquakes and using seismic ways. I will discuss another way, where GPS data from satellites can be used to locate and monitor tectonic faults deeply buried underground in between earthquakes.

Afternoon Plenary Address

Where Parallel Lines Meet: Take a Ride to Infinity on the Projective Plane
Dr. Ed Scheinerman, Johns Hopkins University

Parallel lines do not meet; they have no points in common. And yet, when we look at a long stretch of railroad tracks, the rails appear to be coming closer and closer together, perhaps meeting at the horizon. This feature of how our eyes perceive the world is used by artists to create images that convey a realistic sense of depth. The familiar Euclidean plane doesn't provide a framework for perspective art. When we extend the Euclidean plane with an additional line at infinity, we create a new structure called the projective plane in which parallel lines meet. In this presentation we'll introduce the projective plane, see how to name points and lines with homogeneous coordinates, learn how points and lines are dual to each other, realize that the line at infinity is not special, reveal the topology of the projective plane, and meet the Fano plane – a skinny cousin of the projective plane that has only seven points and seven lines.

Student Presentations

Using Integer Covering Systems to find Riesel and Sierpiński Numbers
Chris Bispels, University of Maryland, Baltimore County

A Riesel number k is an odd integer such that $k2^n - 1$ is composite for all n, the first of which was discovered in 1956 by Hans Riesel. Four years later Wacław Sierpiński defined a similar number, using addition in place of subtraction, called Sierpiński numbers. In this paper, we prove we prove the existence of rep-unit, repdigit, and repnumber Sierpiński and Riesel numbers in different infinite patterns of bases. Additionally, we generalize a previous result on appending a digit repeatedly to an existing Sierpiński number in base 10 to obtain a new Sierpiński number to an infinite number of bases.

Graph Coloring with Restricted Colors
Alejandro Escorcia, Loyola University Maryland

Graph coloring is a well-studied problem in the mathematical field of combinatorics. A (mathematical) graph consists of a set of vertices, some of which are connected by edges. One then asks the question: if there are some number n of available colors, how many ways are there to color the graph? When we vary the values of n, we get a nice formula (which depends on the specific graph) that always gives the answer. This formula is called the chromatic polynomial of a graph, and it is well-known how to calculate it. In this project, we study the number of ways to color a graph when each color is allowed to be used a limited amount of times. It is known that this problem is computationally hard, but we hope to find formulas for the number of colorings in terms of the number of available colors in some special cases.

Topological Quantum Field Theories over Algebraic Field Extensions
Fabian Espinoza, Johns Hopkins University

Two-dimensional Topological Quantum Field Theories (TQFTs) are deeply intertwined with many algebraic structures to encapsulate topological invariants. As per the Cobordism Hypothesis, 2D TQFTs are given by functors from the category of cobordisms to the category of commutative associative Frobenius algebras. In this talk, I will present the construction of two-dimensional TQFTs over algebraic field extensions, emphasizing a shift from fields to Dedekind domains. Frobenius algebras are represented by projective modules, over these domains they are given by fractional ideals within the class field group. The change from fields to Dedekind domains allows for an algebraic shift from dimension 0 to dimension 1, this not only broadens the landscape for TQFTs but also suggests a novel interface with class field theory and arithmetic geometry. In the future, we hope to generalize these notions to higher dimensional varieties/schemes further studying the connections between topological invariants and algebraic geometry.

On Algebraically Defined Edge Colored Graphs
Susanne Goldstein, Oberlin College

An algebraically defined edge-colored graph $(K, \Gamma_{\mathbb{F}}^{c}(f(X,Y)))$ is constructed using a field $\mathbb{F}$, a complete bipartite graph K where each partite set is a copy of $\mathbb{F}^2$, and a function $f(X,Y)$. We denote the vertices of the first partite set by (a_1, a_2) and of the second by $[x_1, x_2]$. We color the edge $(a_1, a_2)\,[x_1, x_2]$ blue if their coordinates satisfy the equation $a_2 + x_2 = f(a_1, x_1)$, otherwise we color the edge red. These graphs are properly connected, meaning that there exists a path with alternating edge colors between any two vertices. We will discuss properties of algebraically defined edge-colored graphs including: proper diameter, the length of the longest minimal proper path length between any pair of vertices; proper girth, the length of the shortest properly colored cycle; and proper vertex cuts, the minimum number of vertices that can be cut to make the graph not properly connected. We motivate our work by a link to incidence geometry.

Vertex Minimal Graphs with Automorphism Groups of Order pq
Alex Holtzman, Towson University

For a finite group G, define $\alpha(G)$ to be the minimum number of vertices among all graphs whose automorphism group is isomorphic to G. This value has been determined for all abelian groups and for various families of nonabelian groups. Define $\alpha(n)$ to be the smallest value of $\alpha(G)$ over all groups G of order n. In this presentation, we calculate the value of $\alpha(pq)$ for most distinct primes p and q. In the process, we construct vertex minimal graphs for the nonabelian semidirect product $\mathbb{Z}_p \ltimes \mathbb{Z}_q$ for $p = 3$ and $p = 5$. We show that $\alpha(\mathbb{Z}_p \ltimes \mathbb{Z}_q) = 2q$, except when $q = 2p + 1$, in which case it is $3q$. For $p = 3$ or 5 and q dividing $2p - 1$, these graphs are vertex minimal for groups of order pq, so $\alpha(pq) = 2q$ for $q > 2p + 1$ and $3q$ otherwise.

A theoretical approach to medical micro-robots
Alex Holtzman, Towson University

Advances in microfluidics are leading to exciting experimental studies of active particles at low Reynolds numbers. In particular, the encapsulation of active particles inside soft matters (droplets or vesicles) holds great promise for biomedical applications such as targeted drug delivery. Various active particles that mimic biological microorganisms have been employed in existing studies. In this presentation, we will discuss the experimental and theoretical progress that has been made in realizing controlled motion of micro-robots in the human body. We will also outline the outstanding challenges that need to be addressed to integrate these life-saving technologies into current healthcare practices.

Developing an Automation Tool to Solve a Logistical Problem Using AMPL and Python
Hai-Hsin Huang, Virginia Military Institute

This project introduces a semi-automated tool to improve the assignment process for 4-H camp activities. The tool collects camper preferences for classes and assigns them to available activities based on those preferences while considering logistical constraints such as activity capacities and age requirements. By applying operations research techniques, we developed a model that significantly reduced assignment time from several hours to just minutes. Data gathered through Google Forms is processed using AMPL and solved with the NEOS CPLEX solver. To enhance efficiency, we integrated Python for data handling, ensuring the system runs with minimal manual intervention. Python's Pandas library facilitates the coordination of inputs and outputs, offering a user-friendly interface. While not fully automated, this approach improves efficiency and accessibility, providing a scalable solution for future 4-H camps and similar organizations to optimize resource allocation.

Applying Deep Learning to Simulate Clark-alpha Turbulence Model
Chenyue Qi, Towson University

This presentation discusses the use of deep learning to simulate the Clark-alpha turbulence model. Starting with an introduction to the Navier-Stokes and Clark-alpha equations, we will cover their applications in fluid dynamics and the advantages of applying deep learning to these computations. A discussion of the numerical results will also be provided.

Twists, Turns, and Topology: Exploring the Fascinating World of Liquid Crystals
Mateusz Ratman, Johns Hopkins University

Liquid crystals are a phase of matter somewhere between ordinary liquids and anisotropic crystals. They are composed of rod-like molecules that tend to collectively align along a given direction. When this order is broken, topological defects appear in the material. In this talk, I will discuss the use of homotopy theory in categorizing defects and explain how topological invariants reveal the stability of various configurations. The concepts of winding number, homotopy groups, and phase transitions will be introduced. In particular, I will show that homotopy theory is able to predict how broken symmetries during phase transitions lead to defect formation. This provides a beautiful example of applying algebraic topology to physical systems. In addition, I will briefly outline the applications of liquid crystals in display technology, biology and cosmology.

Student Presentations (continued)

Spectral Methods in Graph Theory and Finite Topological Spaces

Nicolas Rugo, Johns Hopkins University

Combinatorial properties of graphs (expanding constant, girth, colorability, etc.) can be estimated using asymptotic methods and the eigenvalues of the graph's adjacency matrix. Our aim is to apply and generalize these properties to the study of finite topological spaces.

Inverse Iteration for Laplace Eigenvalue Problems

Ephraim Ruttenberg, University of Maryland, Baltimore County

The Laplacian operator has a discrete, positive spectrum whose smallest ("principal") element is simple, i.e. it has multiplicity 1. This principal eigenvalue is impossible to compute analytically for most domains, but it admits a variational characterization in terms of the Rayleigh quotient. A technique from finite-dimensional linear algebra known as Inverse Iteration can be used to approximate its value, and the application of this method has been well-studied in the presence of Dirichlet boundary conditions. We outline the application of Inverse Iteration to Robin boundary conditions as well as mixed Neumann-Dirichlet boundary conditions.

Mordell's Theorem and Further Developments

Yeju Shin, University of Massachusetts at Amherst

Diophantine equations, equations with two or more variables and integer coefficients, have been studied thoroughly since long ago. One of the most famous studies includes Fermat's Last Theorem, which states that no integer solution larger than two exists in the equation $x^n + y^n = z^n$. This statement led to other questions regarding equations in two variables, including finding the number of solutions. Proved by Mordell, an elliptic curve of the form $y^2 = x^3 + ax^2 + bx$ has a finitely generated abelian group. In this talk, we construct the proof of Mordell's theorem and analyze rank and torsion subgroups of elliptic curves. If time permits, we hope to discuss L-functions and modular forms related to elliptic curves and introduce relevant studies.

Almost Primes of Almost Prime Index

Megan Triplett, Dickinson College

A positive integer is called a k-almost prime if it is a product of k prime numbers, counted with repetition. In this talk we consider j-almost primes of k-almost prime index for given integers $j, k \geq 0$. We establish asymptotic estimates for the counting functions, nth occurrences, and reciprocal sums of such integers.

Underrepresentation in the US House of Representatives: Finding the Probability of Quota Violations in Congressional Apportionment

Tyler Wunder, Johns Hopkins University

The apportionment question asks how to share items that cannot be split up continuously. A relevant fundamental example is congressional apportionment, where United States representatives are apportioned among 50 states based on population. The Balinski-Young theorem states that no perfect apportionment method exists. In this talk, I analyze at the Huntington-Hill method, the method used in the US for congressional apportionment, and analytically find the probability of a quota violation starting at the simple case of three states.