MASON V(irtual) Mid-Atlantic Seminar On Numbers

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Abstracts

Amod Agashe, Florida State University

A generalization of Kronecker's first limit formula with application to zeta functions of number fields

The classical Kronecker's first limit formula gives the polar and constant term in the Laurent expansion of a certain two variable Eisenstein series, which in turn gives the polar and constant term in the Laurent expansion of the zeta function of a quadratic imaginary field. We will recall this formula and give its generalization to more general Eisenstein series and to zeta functions of arbitrary number fields.

Max Alekseyev, George Washington University

Enumeration of Payphone Permutations

People's desire for privacy drives many aspects of their social behavior. One such aspect can be seen at rows of payphones, where people often pick an available payphone most distant from already occupied ones.Assuming that there are n payphones in a row and that n people pick payphones one after another as privately as possible, the resulting assignment of people to payphones defines a permutation, which we will refer to as a payphone permutation. It can be easily seen that not every permutation can be obtained this way. In the present study, we consider different variations of payphone permutations and enumerate them.

Kisan Bhoi, Sambalpur University

Narayana numbers as sum of two repdigits

Repdigits are natural numbers formed by the repetition of a single digit. Diophantine equations involving repdigits and the terms of linear recurrence sequences have been well studied in literature. In this talk we consider Narayana's cows sequence which is a third order liner recurrence sequence originated from a herd of cows and calves problem. We obtain all Narayana numbers expressible as sum of two repdigits. More precisely, we solve the exponential Diophantine equation $N_n = d_1\left(\frac{10^{m_1}-1}{9}\right) + d_2\left(\frac{10^{m_2}-1}{9}\right)$ for some integers $m_1 \leq m_2$ and $d_1, d_2 \in \{1, 2, \ldots, 9\}$ and determine all its positive integer solutions. We find 277 as the largest Narayana number which can be expressed as sum of two repdigits i.e. $277 = 55 + 222$. For the said results, we use lower bounds for linear forms in logarithms (Baker's theory) and a version of Baker-Davenport reduction method in Diophantine approximation.

Damanvir Singh Binner, Simon Fraser University

The Number of Solutions to $ax + by + cz = n$ and its Relation to Quadratic Residues

We develop a formula for the number of non-negative integer solutions (x, y, z) of the equation $ax + by + cz = n$, where a, b, c, and n are given positive integers. The formula leads us to a surprising connection between the number of non-negative integer solutions of the equation $ax + by + cz = n$ and quadratic residues. As a consequence of our work, we are able to prove the equivalence between two fundamental results by Gauss and Sylvester in the nineteenth century that are generally viewed as independent.

Matteo Bordignon, University of New South Wales Canberra

Some new results on the explicit Pólya-Vinogradov inequality

Taken χ a non-principal Dirichlet character modulus q, the Pólya–Vinogradov inequality states that

$$\sum_{n=1}^{N} \chi(n) \ll \sqrt{q} \log q,$$

for any natural number N. In the talk we will present three recent results regarding the explicit version of the Pólya–Vinogradov inequality. We will present the best explicit version for Pólya–Vinogradov for large primitive characters, one for square-free characters (joint work with Bryce Kerr) and at last how this constant is explicitly related to Burgess's bound (joint work with Forrest Francis).

Lisa Cenek, Brittany Gelb, Amherst College, Muhlenberg College

Interactive Theorem Proving with Lean

The Lean proof assistant, a tool for verifying and generating proofs using a computer, is gaining increasing attention among mathematicians. In 2020, Buzzard, Commelin, and Massot formalized a definition of perfectoid spaces, and Kontorovich and Gomes formalized the statement of the Riemann Hypothesis. In this talk, we will introduce and demonstrate interactive theorem proving. We will share our motivation for learning about proof assistants, and we will discuss our process of formalizing results about primitive Pythagorean triples in Lean.

Pengyong Ding, Pennsylvania State University

On a variance associated with the distribution of real sequences in arithmetic progressions

"The talk is composed of two parts. The first part concerns the general result on the following variance associated with the distribution of a real sequence $\{a_n\}$ in arithmetic progressions:

$$V(x, Q) = \sum_{q \le Q} \sum_{a=1}^{q} |A(x; q, a) - f(q, a)M(x)|^2,$$

where $A(x;q,a)$ represents the sum of $\{a_n\}$ in the residue class of a modulo q, and $f(q,a)$ and $M(x)$ approximately reflect the local and global properties of $\{a_n\}$ respectively. We will give a brief history of the problem and provide the standard initial procedure. The second part is an example on calculating the variance $V(x,Q)$ when $a_n = r_3(n)$, the number of ordered representations of n as the sum of three positive cubes, especially on how to calculate the main terms. Finally, some special cases will be discussed at the end of the talk."

Utkal Keshari Dutta, Sambalpur University

Euler-Zagier multiple balancing-like L-functions associated to Dirichlet characters

A natural number m is a balancing number with the balancer r if they are the solution of a simple Diophantine equation $1+2+\cdots+(m-1) = (m+1)+(m+2)+\cdots+(m+r)$. Balancing numbers B_m satisfy the linear recurrence $B_m = 6B_{m-1} - B_{m-2}$ for $m \geq 2$ with initials $B_0 = 0$, $B_1 = 1$, where B_m denotes the m^{th} balancing number. The balancing-like sequence is recursively defined as $x_0 = 0$, $x_1 = 1$ and $x_{m+1} = Ax_m - x_{m-1}$, $m \geq 1$ where $A \in \mathbb{N}_{>2}$. Balancing-like sequence may be considered as generalization of the sequence of natural numbers since the case $A = 2$ describes the sequence of natural numbers. The balancing-like sequence corresponding to $A = 3$ is the sequence of even indexed Fibonacci numbers and the balancing-like sequence corresponding to $A = 6$ is the sequence of balancing numbers.

In this talk, Euler-Zagier multiple balancing-like L-functions associated to Dirichlet characters are introduced as:

$$L_{EZB,k}(s_1,\ldots,s_k \mid \chi_1,\ldots,\chi_k) = \sum_{1\leq m_1<\cdots<m_k} \frac{\chi_1(m_1)}{x_{m_1}^{s_1}} \cdots \frac{\chi_k(m_k)}{x_{m_k}^{s_k}}, \tag{1}$$

where $m_i \in \mathbb{N}$ for $1 \leq i \leq k$ and $\chi_1,\ldots,\chi_k$ are the Dirichlet characters of same modulus $t \in \mathbb{N}_{\geq 2}$ and the analytic continuation of (1) is studied. A complete list of poles and their corresponding residues are calculated. The values of Euler-Zagier multiple balancing-like L-functions associated with Dirichlet characters at negative integer arguments are also examined.

Xander Faber, IDA / Center for Computing Sciences

Totally T-adic Functions of Small Height

A nonzero algebraic number α is totally p-adic if its minimal polynomial over $\mathbb{Q}$ splits completely over $\mathbb{Q}_p$. If α is not a $(p-1)$-st root of unity, then the naive logarithmic height of such an element is uniformly bounded away from zero by an equidistribution result of Bombieri/Zannier or an elementary inequality of Pottmeyer.

In this work, we introduce a geometric analogue. Fix a finite field $\mathbb{F}_q$, and consider the rational function field $\mathbb{F}_q(T)$. An algebraic function that generates a separable extension of $\mathbb{F}_q(T)$ is totally T-adic if its minimal polynomial over $\mathbb{F}_q(T)$ splits completely in the field of Laurent series $\mathbb{F}_q((T))$. We will discuss a lower bound for the height of any nonconstant totally T-adic function, and we will show that functions achieving the lower bound give rise to curious algebraic curves over $\mathbb{F}_q$ with many rational points. We also investigate the limit-infimum of the heights of totally T-adic functions using a dynamical construction.

Hester Graves, IDA / Center for Computing Sciences

The abc conjecture shows that there are infinitely many s-Cullen numbers

Assuming the abc conjecture with $\epsilon = 1/6$, we use elementary methods to show that only finitely many s-Cullen numbers are repunits, aside from two known infinite families. More precisely, only finitely many positive integers s, n, b, and q with $s, b \geq 2$ and $n, q \geq 3$ satisfy

$$C_{s,n} = ns^n + 1 = \frac{b^q - 1}{b - 1}.$$

Shivani Goel, Indraprastha Institute of Information Technology

Moments of Ramanujan Sums over Number Fields

The classical Ramanujan sums are defined as for positive integers m, n as $c_n(m) = \sum_{j=0 (j,n)=1}^{n} e^{\frac{2\pi i j m}{n}} = \sum_{d|n, d|m} d\mu(\frac{n}{d})$. A generalization of Ramanujan sum for number fields was done by Nowak (can check who did it) in which the Ramanujan sum is extended to the ring of integers of algebraic number field. Nowak studied the average order of these sums over quadratic number fields, with respect to both variables m, n by evaluating its first moment. In this talk, we discuss results on the second moment of Ramanujan sums over quadratic field and discuss ideas on extending the proof for first and second moments of Ramanujan sums over cubic fields as well.

Edray Goins, Pomona College

Visualizing Toroidal Belyĭ Pairs

A Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; we may assume these values are $\{0, 1, \infty\}$. A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from 0 to 1. Such graphs can be drawn on the sphere by composing with stereographic projection: $\beta^{-1}\big([0,1]\big) \subseteq \mathbb{P}^1(\mathbb{C}) \simeq S^2(\mathbb{R})$.

Replacing $\mathbb{P}^1$ with an elliptic curve E, there is a similar definition of a Belyĭ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$. The corresponding Dessin d'Enfant can be drawn on the torus by composing with an elliptic logarithm: $\beta^{-1}\big([0,1]\big) \subseteq E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$. In this project, we use the open source `Sage` to write code which takes an elliptic curve E and a Belyĭ map β to return the Dessin d'Enfant of this map – both in two and three dimensions. We focus on several examples of Belyĭ maps which appear in the L-Series and Modular Forms Database (LMFDB).

Jon Grantham, IDA/CCS

Reversed Sum-Product Pairs

We determine all pairs of positive integers (a, b) such that $a + b$ and $a \times b$ have the same decimal digits in reverse order:

$$(2, 2), (9, 9), (3, 24), (2, 47), (2, 497), (2, 4997), (2, 49997), \ldots$$

We use deterministic finite automata to describe our approach, which naturally extends to all other numerical bases.

Hayder Hashim, University of Debrecen

Diophantine equations involving linear recurrence sequences

In this talk, we focus on our results in which we extended the result of Tengely in which he determined all the solutions (x, n) with $x \geq 2$ for the Diophantine equation

$$\frac{1}{U_n(P,Q)} = \sum_{k=0}^{\infty} \frac{U_k(P,Q)}{x^{k+1}},$$

for certain nonzero parameters (P, Q) and $\{U_n(P, Q)\}$ denotes the Lucas sequence of the first kind that is defined by

$$U_0 = 0, U_1 = 1 \text{ and } U_n(P,Q) = PU_{n-1} - QU_{n-2}, \text{ if } n \geq 2.$$

We first studied the solutions of the equation

$$\frac{1}{U_n(P_2,Q_2)} = \sum_{k=0}^{\infty} \frac{U_k(P_1,Q_1)}{x^{k+1}},$$

for certain given pairs $(P_1, Q_1) \neq (P_2, Q_2)$. We also considered the equation

$$\sum_{k=0}^{\infty} \frac{U_k(P,Q)}{x^{k+1}} = \sum_{k=0}^{\infty} \frac{R_k}{y^{k+1}},$$

where $\{R_n\}$ is a ternary linear recurrence sequence represented by the Tribonacci sequence $\{T_n\}$ or Berstel's sequence $\{B_n\}$, that are respectively defined by

$$T_0 = T_1 = 0, T_2 = 1, \quad T_{n+3} = T_{n+2} + T_{n+1} + T_n,$$
$$B_0 = B_1 = 0, B_2 = 1, \quad B_{n+3} = 2B_{n+2} - 4B_{n+1} + 4B_n$$

for $n \geq 0$. We also provided general results related to the integer solutions (x, y) of the equations

$$\sum_{k=0}^{\infty} \frac{U_k(P_1,Q_1)}{x^{k+1}} = \sum_{k=0}^{\infty} \frac{U_k(P_2,Q_2)}{y^{k+1}},$$

where the pairs $(P_1, Q_1) \neq (P_2, Q_2)$, and

$$\sum_{k=0}^{\infty} \frac{T_k(a_2, a_1, a_0)}{x^{k+1}} = \sum_{k=0}^{\infty} \frac{T_k(b_2, b_1, b_0)}{y^{k+1}},$$

with $(a_2, a_1, a_0) \neq (b_2, b_1, b_0)$ and T_n denotes the general term of the generalized Tribonacci sequence defined by $T_0(p, q, r) = T_1(p, q, r) = 0, T_2(p, q, r) = 1$, and

$$T_n(p, q, r) = pT_{n-1}(p, q, r) + qT_{n-2}(p, q, r) + rT_{n-3}(p, q, r) \text{ if } n \geq 3.$$

Then we applied the results to completely solve such equations with certain parameters. This is a joint work with my supervisor (Szabolcs Tengely from University of Debrecen).

Edna Jones, Rutgers University

An Asymptotic Local-Global Principle for Integral Kleinian Sphere Packings

We will discuss an asymptotic local-global principle for certain integral Kleinian sphere packings. Examples of Kleinian sphere packings include Apollonian circle packings and Soddy sphere packings. Sometimes each sphere in a Kleinian sphere packing has a bend (1/radius) that is an integer. When all the bends are integral, which integers appear as bends? For certain Kleinian sphere packings, we expect that every sufficiently large integer locally represented as a bend of the packing is a bend of the packing. We will discuss ongoing work towards proving this for certain Kleinian sphere packings. This work uses quadratic forms, the circle method, spectral theory, and expander graphs.

Steven Jin, University of Maryland

An Elliptic Curve Analogue of Pillai's Bound on Least Primitive Roots

A classically studied problem involves bounding from above and below the least primitive root of a prime p, denoted $r(p)$. From below, a classical bound due to Pillai states that for infinitely many primes, we have $r(p) > \log \log p$. His approach relies on showing that enough small elements of $(Z/pZ)^{\times}$ are quadratic residues and hence not generators of $(Z/pZ)^{\times}$. The key ingredient used is quadratic reciprocity for $n = 2$. Pillai's bound has since been improved by Fridlander and Salie to $C \log p$. In this talk, we discuss an elliptic curve analogue of Pillai's bound, where we bound from below the least generator of the point group of an elliptic curve over a finite field. Our proof relies crucially on an effective Chebotarev estimate on the smallest splitting prime ideal in a number field.

Christopher Keyes, Emory University

An upper bound for the number of arithmetical structures on a graph

Let G be a connected undirected graph on n vertices with no loops but possibly multiedges. Given an arithmetical structure $(\mathbf{r}, \mathbf{d})$ on G , we describe a construction which associates to it a graph G' on $n-1$ vertices and an arithmetical structure $(\mathbf{r}', \mathbf{d}')$ on G' . By iterating this construction, we derive an upper bound for the number of arithmetical structures on G depending only on the number of vertices and edges of G. In the specific case of complete graphs, possibly with multiple edges, we refine and compare our upper bounds to those arising from counting unit fraction representations. This is joint work with Tomer Reiter.

Matthew Litman, University of California, Davis

Distinct Residues of Lucas Polynomials over $\mathbb{F}_p$

Given a polynomial with integral coefficients, one can inquire about the possible residues it can take in its image modulo a prime p. The sum over these residues can sometimes be computed independently of the choice of prime p; for example, Gauss showed that the sum over quadratic residues vanishes modulo a prime. In this talk, we will provide a closed form for the sum over distinct residues in the image of Lucas polynomials of arbitrary degree over all primes, and prove a complete characterization of the size of the image set. Our result provides the first non-trivial classification of such a sum for a family of polynomials of unbounded degree. This talk is based on joint work with Thomas Brazelton, Joshua Harrington, and Tony W.H. Wong.

Michael Mossinghoff, Center for Communications Research

Wolstenholme and Vandiver primes

In 1862 the Reverend J. Wolstenholme established a number of congruences modulo powers of primes for certain expressions involving binomial coefficients, harmonic numbers, and related quantities. A prime for which one of these congruences becomes a "super-congruence" (holding modulo a higher power of the prime) is known as a *Wolstenholme prime*. While it is conjectured that infinitely many such primes exist, very few are known. Wolstenholme primes can also be defined by using Bernoulli numbers, which arise widely in number theory, for example in the study of regular primes. A *Vandiver prime* can be defined in an analogous way, but using Euler numbers in place of Bernoulli numbers; these primes arose in the study of Fermat's Last Theorem. Here again it is conjectured that infinitely many exist, but only a few are known. We describe some new and extensive searches for Wolstenholme and Vandiver primes. To power this, we develop a number of new congruences for Bernoulli numbers and for Euler numbers that are favorable for computation, and we implement some highly parallel searches using GPUs.

Shashika Petta Mestrige, Louisiana State University

Congruences for some partition functions

Ramanujan, Watson, Atkin, and Gordon used modular functions and modular equations to prove remarkable congruences of the partition function $p(n)$ and multi-partitions $p_k(n)$.

By extending their ideas, we proved the congruences for two parameter family of partitions $p_{[1^c\ell^d]}(n)$ modulo powers of primes $5 \leq \ell \leq 17$. We define these partitions by

$$\sum_{n=0}^{\infty} p_{[1^c\ell^d]}(n)q^n = \prod_{n=1}^{\infty} \frac{1}{(1-q^n)^c(1-q^{\ell n})^d}.$$

Then we used them to derive congruences and incongruences for ℓ-regular partitions, ℓ-core partitions, and ℓ-colored generalized Frobenius partitions.

Then we investigated the congruences modulo arbitrary prime powers by studying a ℓ-adic module associated to the partitions $p_{[1^c\ell^d]}(n)$. Our work has been inspired by the work of Matthew Boylan and John Webb.

Evangelos Nastas, SUNY

A Generalization of the Ramanujan-Nagell Theorem

This talk is devoted to a new generalization of the Ramanujan-Nagell equation. More precisely, the main reason for this project is to shed light on the following equation $x^2+p = 2^n$, where $p = 2^k - 1$, and $\forall k \in \mathbb{N}^+$. For $K = 3$ the aforementioned equation takes the form $x^2 + 7 = 2^n$, which has already been solved by Nagell proving Ramanujan's conjecture that its only solutions are:

$$x = \pm 1 \pm 3 \pm 5 \pm 12 \pm 181$$

$$n = \ 3, 4, 5, 7, 15$$

by using Nagell's proof and by generalizing it we are looking for the solutions of $x^2 + p = 2^n$.

Zeraoulia Rafik, University of Batna2

On congruence of the iterated form $\sigma^k(m) = 0 \bmod m$

"Inspired by the question of Graeme L. Cohen and Herman J. J. te Riele , The Authors of [**?**] who they investigated a question :Given n is there an integer k for which $\sigma^k(n) = 0 \bmod n$? They did this in a 1995 paper and asserted through computation that the answer was yes for $n \leq 400$,The aim of this paper is to give a negative answer to the reverse question of Graeme L. Cohen and Herman J. J. te Riele such that we shall prove that there is no fixed integer n for which $\sigma^k(n) = 0 \bmod n$ for all iteration k of sum divisor function using H.Lenstra problem for aliquot sequence at the same time we prove that there exist an integer n satisfies $\sigma^k(n) = 0 \bmod n$ for all odd iteration k rather than that we shall prove that if n is multiperfect with L the lcm (the least common multiple) of 1+each exponents in the prime factorization of n , and with L prime, then $n = 6$.we conclude our paper with open conjecture after attempting to show that 6 is the only integer satisfy periodicity with small period dividing L"

Anwesh Ray, University of British Columbia

Arithmetic statistics and Iwasawa theory

In this talk, we examine the average behaviour of the Iwasawa invariants for the Selmer groups of elliptic curves, setting out new directions in arithmetic statistics and Iwasawa theory.

Wendell Ressler, Franklin & Marshall College

Conjugacy classes and rational period functions for the Hecke groups

We describe a correspondence between conjugacy classes of Hecke groups and irreducible systems of poles of rational period functions for automorphic integrals on the same groups. We use this relationship to determine properties of pole sets and to construct new families of rational period functions.

Bolbachan Vasily, Higher School of Economics, Moscow

Functional Equations for Elliptic Dilogarithm

The Elliptic dilogarithm was defined by Spencer Bloch. Functional equations for the classical dilogarithm play an important role in the applications of this remarkable function.

Let E be an elliptic curve over an algebraically closed field of characteristic 0. The *pre-Bloch* group is defined as the quotient of the free abelian group generated by the non-zero rational functions on E by the subgroup generated by the so-called Abel five-term relations. I will explain that this group can generated by the functions of degree not higher than 3.

Bloch has shown that any element of the pre-Bloch group gives a (so-called elliptic Bloch) relation between the values of the Elliptic dilogarithm. I will show that any elliptic Bloch relation can be reduced to the antisymmetry relation and the elliptic Bloch relations for the functions of degree 3.

Ian Whitehead, Swarthmore College

Apollonian Packings and Kac-Moody Root Systems

Fix four mutually tangent circles in the plane. In the spaces between circles, fill in additional tangent circles. By repeating this process ad infinitum, on smaller and smaller scales, we obtain an Apollonian circle packing. I will define a four-variable generating function for curvatures that appear in a given packing. This generating function is essentially a character for a rank four indefinite Kac-Moody root system. I will relate this generating function to certain automorphic forms, including theta functions on SL(2) and a Siegel modular form on Sp(4). And I will describe its domain of convergence, the Tits cone of the root system, which inherits the intricate geometry of Apollonian packings.

Shaoyun Yi, University of South Carolina

On counting cuspidal automorphic representations of GSp(4)

There are some well-known classical equidistribution results like Sato-Tate conjecture for elliptic curves and equidistribution of Hecke eigenvalues of elliptic cusp forms. In this talk, we will discuss a similar equidistribution result for a family of cuspidal automorphic representations for GSp(4). We formulate our theorem explicitly in terms of the number of cuspidal automorphic representations for GSp(4) with certain conditions at the local places. To count the number of these cuspidal automorphic representations, we will explore the connection between Siegel cusp forms of degree 2 and cuspidal automorphic representations of GSp(4). This is a joint work with Manami Roy and Ralf Schmidt.