Mid-Atlantic Seminar On Numbers IV

Saturday and Sunday, March 7 & 8, 2020

Gettysburg College

List of Abstracts

Lindsay Dever, Bryn Mawr College

SATURDAY 10:00 – 10:20, SCIENCE CENTER 200

Prime geodesics on compact, hyperbolic 3-manifolds

Every hyperbolic 3-manifold arises as the quotient of $SL_2(\mathbb{C})$ by a discrete, torsion-free subgroup. Compact arithmetic hyperbolic 3-manifolds and the corresponding Kleinian groups encode arithmetic information about quaternion algebras. There is a deep connection, made explicit by the trace formula, between spectral information about Laplacian eigenvalues and the set of closed geodesics on the manifold. Prime geodesic theorems provide an asymptotic count for primitive geodesics, analogous to the famed Prime Number Theorem. In this talk, I will discuss the geometry and spectra of hyperbolic 3-manifolds and present new results about the distribution of prime geodesics on compact, hyperbolic 3-manifolds.

Helen G. Grundman, Bryn Mawr College

Plenary Talk

SATURDAY 9:00 – 9:50, SCIENCE CENTER 200

Joy & Happiness: Digital Sums and Variations

A harshad number ("harshad" is Sanskrit for "joy-giver") is a positive integer that is an integer multiple of the sum of its digits. A happy number is a positive integer such that iteratively taking the sum of the squares of its digits eventually yields the number 1. I will discuss research on these numbers and a wide range of variations, surveying results and open questions, while describing some of the key methods of proof.

Joshua Harrington, Cedar Crest College

SATURDAY 11:40 – 12:00, SCIENCE CENTER 202

Classifying Super-totient Numbers

A positive integer n is called a super-totient number if the set of positive integers less than n and relatively prime to n can be partitioned in two sets of equal sum. In this talk we present a complete classification of all super-totient numbers and discuss a generalization of this concept.

Steven J. Miller, Williams College

Plenary Talk

SATURDAY 2:00 – 2:50, SCIENCE CENTER 200

From the Manhattan Project to Elliptic Curves

Physicists developed Random Matrix Theory (RMT) in the 1950s to explain the energy levels of heavy nuclei. A fortuitous meeting over tea at the Institute in the 1970s revealed that similar answers are found for zeros of L-functions, and since then RMT has been used to model their behavior. The distribution of these zeros is intimately connected to many problems in number theory, from how rapidly the number of primes less than X grows to the class number problem to the bias of primes to be congruent to 3 mod 4 and not 1 mod 4. We report on recent progress on understanding the zeros near the central point, emphasizing the advantages of some new perspectives and models. We end with a discussion of modular forms, especially elliptic curves. We'll mix theory and experiment and see some surprisingly results, which lead us to conjecture that a new random matrix ensemble correctly models the small conductor behavior.

Russell Jay Hendel, Towson University

SUNDAY 10:00 – 10:20, SCIENCE CENTER 200

Recursive Triangles embedded in Families of Recursive Sequences

Fix positive integers t and m. Let $k \geq t+3$ be a parameter varying over the positive integers.

Recursively define a triangle $T = \{T_{i,j}\}_{i \geq 1}$ as follows: $\langle T_{1,1}, T_{1,2} \rangle = \langle 1, -1 \rangle$, with $T_{1,q} = 0$, if either $q < 1$ or $q > 2$. For $p \geq 1$, recursively define $T_{p+1,q} = -(m-1)T_{p,q} + (m-1)T_{p,q-1} + 2T_{p,q-t-1} - T_{p,q-t-2}$. (Angle brackets indicates ordered sequences)

When the context is clear, we let the phrase p-th row of the triangle refer to $T_p = \langle T_{p,1}, T_{p,2}, \ldots, T_{p,2+(p-1)(t+2)} \rangle$, even though technically the rows of the triangle are doubly infinite. It is easy to show that $T_{p,1}$ and $T_{p,2+(p-1)(t+2)}$ are non-zero.

We consider an infinite family of recursions, with the k-th recursion, $k \geq t+3$, given by $G_n = G_{n-k} - \sum_{i=1}^{k-1} G_{n-k+i} - (m-1)G_{n-k+(t+1)}$, with initial conditions, $G_1 = 1$, $G_i = 0, 2 \leq i \leq k$.

The main theorem asserts that given arbitrary $n \geq 1$, then for all sufficiently large $k \geq k(n,t)$,

$$\langle G_1, G_2, \ldots, G_{k+n(k-t-1)} \rangle = \langle 1, 0^{k-1}, T_1, 0^{z_1}, T_2, 0^{z_2}, \ldots, T_n, 0^{z_n} \rangle,$$

with 0^z indicating a sequence of z zeroes, and where the $z_i, i = 1, 2, \ldots, n$ are non-negative integers such that after the initial-conditions' block, the length of each block $\langle T_m, 0^{z_m} \rangle$ is $k-t-1$.

The theorem heuristically states that the recursive triangle laid out row by row, $(T_1, T_2, \dots)$ is embedded in the limit of the order-k recursive sequences with boundaries of 0s delimiting the triangle rows. Equivalently the main theorem states that (for k sufficiently large) the $p-th$ block, $\langle T_p, 0^{z_p} \rangle, p \geq 1$, can be simultaneously derived from either the order-k recursion or the triangle recursion.

The triangles for $m = 2, 3, ...$ and arbitrary t do not show up in any previous OEIS sequences implying that these sequences and results are new; they were recently published in new OEIS sequence A332636. The idea of triangles embedded in limits of recursive families is also new. However, for $m = 1, t = 1$ the triangle (with zeroes removed) laid out row by row in this presentation is identical to sequence OEIS 118800 which uses a variety of methods to describe the sequence including Riordan arrays, several matrix approaches, row polynomials, Chebychev polynomials, and the DELTA operator approach (defined in OEIS A084938). This presentation provides a simple proof based on the characteristic polynomials of the underlying recursions.

Michael Knapp, Loyola University Maryland

SUNDAY 10:30 – 10:50, SCIENCE CENTER 200

Inhomogeneous Additive Equations

Let k and n be distinct positive integers. Define the function $\Delta^*(k,n)$ to be the smallest number s which guarantees that the equation

$$a_1x_1^k + \cdots + a_sx_s^k + b_1y_1^n + \cdots + b_sy_s^n = 0$$

has nontrivial solutions in each of the p-adic fields $\mathbb{Q}_p$, regardless of the rational integer coefficients. This function is related to the well-studied Γ^*-function of Davenport & Lewis, in which there is only one degree. In this talk, we will give some introductory results about $\Delta^*(k,n)$, including a sharp upper bound for its values.

Angel Kumchev, Towson University

SUNDAY 10:00 – 10:20, SCIENCE CENTER 202

Diminishing ranges for Diophantine inequalities

Consider a diagonal form $F(\mathbf{x}) = \lambda_1 x_1^k + \cdots + \lambda_s x_s^k$, where $\lambda_1, \ldots, \lambda_s$ are nonzero reals, not all of the same sign and with at least one ratio λ_i/λ_j irrational. One expects that the values of this form at integer points $\mathbf{x}$ to be dense in $\mathbb{R}$, at least when s is sufficiently large. Moreover, one expects such results when the variables are restricted to special sets of integers. In particular, the study of the distribution of the values $F(\mathbf{p})$ at points $\mathbf{p}$ with prime coordinates is akin to the Waring-Goldbach problem. Let $D(k)$ denotes the least s for which the values $F(\mathbf{p})$ are dense in the real line. We are interested in upper bounds for $D(k)$ that match (or are close to) the upper bound for the analogous function $H(k)$ studied in the Waring-Goldbach problem. The best bounds for $H(k)$ rely on Davenport's method of diminishing ranges—a collection of bounds for counts of equations with size-restricted variables. Some, but not all, of these bounds have analogues for Diophantine inequalities, and as result the best known bounds for $D(k)$ lag behind the best bounds for $H(k)$. We will discuss some progress towards rectifying this situation.

Hiram López, Cleveland State University

Saturday 4:40 – 5:00, Science Center 202

Applications of the trace function and vanishing ideals to storage systems

In this talk we will study families of linear codes that are locally recoverable codes designed for distributed and cloud storage systems. In this context, locally means that only a few servers of the storage system have to be accessed to retrieve the information for a certain server. Then we will see how algebraic concepts, like the trace function, or the vanishing ideal, may play an important role on this sort of applications.

Amita Malik, Rutgers University

Plenary Talk

SUNDAY 9:00 – 9:50, SCIENCE CENTER 200

Zeros of derivatives of the completed Riemann zeta function

Riemann Hypothesis implies the Riemann Hypothesis for higher order derivatives of the completed Riemann zeta function. The gap distribution of the zeros of these derivatives would provide insight into the existential question of the Landau-Siegel zero. In this talk, we discuss the vertical distribution of these zeros and establish their uniform distribution modulo one. Moreover, we prove that 100% of the zeros of combinations of these derivatives lie on the critical line even though there is no Euler product in this case.

Gretchen Matthews, Virginia Tech

SATURDAY 11:40 – 12:00, SCIENCE CENTER 200

Explicit non-special divisors of small degree and applications

Consider a curve X of genus g over a finite field. Recall that a divisor A is non-special if its dimension is given by $\deg(A)+1-g$ and special otherwise. Divisors of large degree are necessarily non-special, and divisors of very small degree are necessarily special. In this talk, we share recent work on divisors of small degree which are also non-special and discuss applications to coding theory and cryptography.

James McLaughlin, West Chester University, PA

SATURDAY 4:40 – 5:00, SCIENCE CENTER 200

New Infinite q-Product Expansions with Vanishing Coefficients

Motivated by results of Hirschhorn, Tang, and Baruah and Kaur on vanishing coefficients (in arithmetic progressions) in a new class of infinite product which have appeared recently, we further examine such infinite products, and find that many such results on vanishing coefficients may be grouped into families. Recall that

$$(a;q)_\infty := (1-a)(1-aq)(1-aq^2)\cdots$$
$$(a_1,\ldots,a_j;q)_\infty := (a_1;q)_\infty\cdots(a_j;q)_\infty$$

For example, one result proven is that if $b \in \{1,2,\ldots,9,10\}$ and the sequence $\{r_n\}$ is defined by

$$(q^{8b},q^{11-8b};q^{11})^3_\infty(q^{11-b},q^{11+b};q^{22})_\infty =: \sum_{n=-756}^{\infty} r_n q^n.$$

then $r_{11n+6b^2+b} = 0$ for all n. Further, if $b \in \{1,3,5,7,9\}$, then $r_{11n+4b^2+b} = 0$ for all n also. Each particular value of b gives a specific result, such as the following (for $b = 1$): if the sequences $\{a_n\}$ is defined by

$$\sum_{n=0}^{\infty} a_n q^n := (q^3,q^8;q^{11})^3_\infty(q^{10},q^{12};q^{22})_\infty,$$

then $a_{11n+5} = a_{11n+7} = 0$.

Nathan McNew, Towson University

SATURDAY 11:10 – 11:30, SCIENCE CENTER 202

Counting pattern avoiding integer partitions

A partition α contains another partition (or pattern) μ if the Ferrers board for μ is attainable from α under removal of rows and columns. We say α avoids μ if it does not contain μ. We count the number of partitions of n avoiding a fixed pattern μ, in terms of generating functions and their asymptotic growth rates.

Nathan Ryan, Bucknell University

SUNDAY 10:00 – 10:20, SCIENCE CENTER 202

Congruences satisfied by eta-quotients

The values of the partition function, and more generally the Fourier coefficients of many modular forms, are known to satisfy certain congruences. Results given by Ahlgren and Ono for the partition function and by Treneer for more general Fourier coefficients state the existence of infinitely many families of congruences. In this article we give an algorithm for computing explicit instances of such congruences for eta-quotients. We illustrate our method with a few examples.

Zachary Scherr, Susquehanna University

SUNDAY 11:00 – 11:20, SCIENCE CENTER 200

Integral Polynomial Pell Equations

In his work on the Pell equation, Euler discovered several interesting polynomial identities included among them that $(2n^2+1)^2-(n^2+1)(2n)^2=1$ for every n. Motivated by Euler's examples, one can ask whether it is possible to classify all such identities. In particular, for which polynomials $d(x)\in\mathbb{Z}[x]$ do there exist non-trivial solutions to $f(x)^2-d(x)g(x)^2=1$ with $f(x),g(x)\in\mathbb{Z}[x]$? Yokota and Webb classified all such quadratic $d(x)$ and asked about the situation with $d(x)$ of degree at least 4. In this talk we'll classify all monic, quartic, square-free $d(x)$ which give rise to non-trivial solutions to Pell's equation. In particular we'll show that there are exactly two infinite families of such $d(x)$. The resolution of this problem is intimately tied with work by Mazur and Kubert on rational torsion points on elliptic curves.

Dane Skabelund, Virginia Tech

SATURDAY 4:00 – 4:20, SCIENCE CENTER 200

On the irreducibility of the non-reciprocal part of polynomials of the form $f(x)x^n + g(x)$

In recent work, Sawin, Shusterman and Stoll study polynomials of the form $f(x)x^n + g(x)$. Under certain conditions on f and g, they provide a lower bound on n such that the non-reciprocal part of such a polynomial is irreducible. This bound is exponential in the sum of squares of the coefficients of f and g, with base 2. In this talk, I will describe an improvement on this bound, which effectively replaces the base 2 with the golden ratio. This is joint work with Michael Filaseta, Huixi Li, and Frank Patane.

Emerald Stacy, Washington College

SATURDAY 4:00 – 4:20, SCIENCE CENTER 202

A Congruence Condition for Abelian Totally p-adic Numbers of Small Height

Given an algebraic number α, the height of α gives a measure of how arithmetically complicated α is. We say an algebraic number is totally p-adic if its minimal polynomial splits completely over $\mathbb{Q}_p$. In this talk, we restrict our gaze to algebraic numbers that live in abelian extensions of $\mathbb{Q}$.

Robert Vaughan, Penn State University

SATURDAY 10:00 – 10:20, SCIENCE CENTER 202

On some questions of partitio numerorum: Tres cubi

We discuss some problems concerning sums of cubes of non-negative integers, and in particular sums of three cubes.

Daniel White, Bryn Mawr College

SUNDAY 11:00 – 11:20, SCIENCE CENTER 202

Extreme values of *L*-functions

Soundararajan's resonance method provides insight on the extremal behavior of the zeta function along the critical line and the last decade has seen a number of adaptations of this method for application to various families of *L*-functions. In this talk, a conceptual illustration of the resonance method will be given and new preliminary results on *L*-functions over number fields will be presented.

Michael Wills, Virginia Tech

SATURDAY 11:10 – 11:30, SCIENCE CENTER 200

Weierstrass semigroups of triples of points on the Hermitian curve

The Weierstrass semigroup of a point on a curve is a classically studied numerical semigroup, the cardinality of which is specified by the Weierstrass Gap Theorem. In the 1980's the Weierstrass semigroup of a pair of points was defined, and more recently there has been work on semigroups of m-tuples of points. In this talk, we share new results on the Weierstrass semigroup of triples of points on the Hermitian curve. The study of these objects relies on the Riemann-Roch Theorem and discrepancies of two points, introduced by Duursma and Park in 2012. This is joint work with Gretchen Matthews and Dane Skabelund.

Wing Hong Tony Wong, Kutztown University of Pennsylvania

SATURDAY 10:30 – 10:50, SCIENCE CENTER 202

Integer partitions with equal products

An ordered triple (s, p, n) is called admissible if there exist two different multisets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ such that X and Y share the same sum s, the same product p, and the same size n. We first count the number of n such that (s, p, n) are admissible for a fixed s. We also fully characterize the values p such that (s, p, n) is admissible. This project is also related to John Conway's wizard puzzle from the 1960s.

Jeff Yelton, Emory University

SATURDAY 10:30 – 10:50, SCIENCE CENTER 200

Abelian surfaces over number fields locally having order-ℓ^2 subgroups

Suppose that we are given an integer $m \geq 2$ and an abelian variety A over a number field K which satisfies the following property: for a density-1 set of primes $\mathfrak{p}$ of K, the group of points of reduction of A modulo $\mathfrak{p}$ over the residue field has order divisible by m. Lang posed the question of whether this implies that there exists an abelian variety A' which is K-isogenous to A such that $A'(K)$ has order divisible by m. N. Katz has shown that the answer is affirmative when A has dimension ≤ 2 and $m = \ell$ is a prime. I will present new results from joint work with John Cullinan treating the case where A has dimension 2 and $m = \ell^2$ is the square of a prime, where we show that in this case the answer is negative and provide a classification of each type of counterexample that occurs for different primes ℓ.