MASON II: Second Mid-Atlantic Seminar On Numbers

April 7–8, 2018

Abstracts

Max Alekseyev, George Washington University

On Partitions into Squares of Distinct Integers Whose Reciprocals Sum to 1

In 1963, Graham proved that all integers greater than 77 (but not 77 itself) can be partitioned into distinct positive integers whose reciprocals sum to 1. He further conjectured that any sufficiently large integer can be partitioned into squares of distinct positive integers whose reciprocals sum to 1. We establish the exact bound for existence of such partitions. Namely, we prove that 8542 is the largest integer that cannot be partitioned this way.

Michael Bush, Washington and Lee University

Non-abelian Generalizations of the Cohen-Lenstra Heuristics

In the last few years, my collaborators and I have proposed a non-abelian version of the Cohen-Lenstra heuristics for both real and imaginary quadratic fields in which one replaces the p-part of the class group (p odd prime) with the Galois group of the maximal unramified p-extension of the base field. One big difference when making this switch is that the latter objects can be infinite while the former are always finite. Our conjectures originally only dealt with the frequency of occurrence of finite p-groups and certain special finite quotients of the infinite pro-p groups that arise. After giving an overview, I'll discuss some recent developments including an extension covering infinite groups. This is joint work with Nigel Boston and Farshid Hajir.

Eva Goedhart, Lebanon Valley College

Solving Some Diophantine Equations with Linear Forms in Logarithms

After a quick history and a couple definitions, I will present a friendly version of how to use linear forms in logarithms to help solve certain families of Diophantine equations.

Jon Grantham, IDA/CCS

Yet Another Conjecture of Goldbach: Preliminary Results

Let A be the set of numbers a for which a^2+1 is prime. Goldbach conjectured that every $a \in A$ $(a > 1)$ can be written in the form $a = b + c$, for $b, c \in A$. We present computational verification of the conjecture up to 2.5×10^{14} as well as conditional results related to it.

Joshua Harrington, Cedar Crest College

Odd Coverings of Subsets of the Integers

Let S be a set of integers. A covering system of S is a finite collection of congruences such that every integer in the set satisfies at least one of the congruences in the collection. An odd covering of S is a covering system such that all moduli are distinct, odd, and greater than 1. Filaseta and Harvey recently investigated the existence of odd coverings of certain subsets of the integers. In this talk we extend this investigation and address a question of Filaseta and Harvey.

Russell Hendel, Towson University

Further Exploration of Tagiuri Generated Families of Fibonacci Identities

This paper continues the work on the Tagiuri Generating Method (TGM) for production of Fibonacci identities recently introduced at the Caen Fibonacci conference.

Step 1: Let $q \geq 1$ be a positive integer.
Step 2: Let $P = \prod_{i=1}^{4q} F_{n+a_i}$, with the a_i parameters.
Step 3: Let $P = P_j, 0 \leq j \leq 6q$, be $6q+1$ identical copies of P.
Step 4: The following *start identity* is trivially true.

$$2qP_0 = \sum_{j=1}^{4q} P_j - \sum_{j=4q+1}^{6q} P_j.$$

Step 5: Define the infinite string,

$$S = \{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4);(5,6),(5,7),(5,8),(6,7),(6,8),(7,8);\dots\}.$$

Step 6: We *apply* the Tagiuri identity, $F_{n+x}F_{n+y} = F_nF_{n+x+y} + (-1)^n F_xF_y$, using the first $4q$ components in S. In other words, we apply Tagiuri with $x = a_1, y = a_2$ in P_1, and then apply Tagiuri with $x = a_1, y = a_3$ in P_2, and then apply Tagiuri with $x = a_1, y = a_4$ in P_3 ....

Step 7: Finally, we make the following substitutions: $a_1 = -2q, a_2 = -(2q-1), \dots, a_q = -1, a_{q+1} = 1, a_{q+2} = 2, \dots, a_{2q} = 2q$. This sequence of seven steps gives us a Fibonacci identity in parameter n. The sequence of identities, $I_1, I_2, \dots,$ is an infinite family of identities. In general, nothing can be said about these identities. However,

Step 8: we can expand all right sides of these identities without combining like terms.

Step 9: We can then count with multiplicity occurrences of indices $n \pm z$ but ignore coefficients and signs. We can define for each I_r the index histogram of the indices as a function, H_r from indices $n \pm z$ to $\mathbb{N}$.

The following main theorem continues a set of theoretic results on other Tagiuri-Generated families of identities:

Theorem: *The cardinality of the range of each H_r is bounded by 7.*

We present further theorems completely describing the H_r. Some open problems and directions of future research are discussed.

Thomas Hulse, Morgan State University

Some Applications of Shifted Sums

It has long been known that shifted sums of Fourier coefficients of automorphic forms can be obtained through a Rankin-Selberg-style untiling with a real-analytic Poincar series. However, these shifted sums can, in turn, be used to produce unstudied meromorphic continuations of other objects related to problems of broad interest in number theory. This talk will demonstrate several applications of these constructions, relating to bounds of partial sums of Fourier coefficients, information about the generalized Gauss Circle Problem, and the Congruent Number Problem.

Mike Knapp, Loyola University of Maryland

Integral Values of Generating Functions of Fibonacci-like Sequences

In this talk, we study sequences defined recursively as follows. Fix a positive integer $a \geq 3$, and define the sequence G_n by

$$G_0 = a, \quad G_1 = 1, \quad \text{and} \quad G_n = G_{n-1} + G_{n-2} \text{ for } n \in \mathbb{N}.$$

Let $G(x)$ be the generating function for this sequence. In this talk, we study the problem of finding the rational values of x such that $G(x)$ is an integer. We give a method which, for any given value of a, reduces the problem to a finite computation, and also give several families of solutions which work for all values of a.

⋆ **Alex Konotorovich**, Rutgers University

Circles and Numbers

People have long been fascinated by the beauty of iterated symmetry. We will discuss a long list of mathematicians and scientists who have delved into questions at the intersection of geometry, number theory, and chaotic dynamics.

⋆ **Wen-Ching Winnie Li**, Penn State University

Distribution of Primes

The distribution of prime numbers has been one of the central topics in number theory. It has a deep connection with the zeros of the Riemann zeta function. The concept of "primes" also arises in other context. For example, in a compact Riemann surface, as introduced by Selberg, primitive closed geodesic cycles play the role of primes; while in a finite quotient of a finite-dimensional building, for each positive dimension, there are primes of similar nature. In this talk we shall discuss the distributions of such primes and their connections with the analytic behavior of the associated zeta and L-functions.

Amita Malik, Rutgers University

Restricted Partitions Analysis

The asymptotics for the partition function were studied by Hardy and Ramanujan in the early 1900. The first result pertaining the parity of this function was proved only in 1959. In this talk, we discuss these questions for certain types of restricted functions, and see how one can make use of the divisor function to obtain some non-trivial lower bounds for the parity in question.

⋆ **Jennifer Park**, University of Michigan

Chabauty-Coleman Method for Rational Points on Varieties

Faltings' theorem states that there are finitely many rational points on curves of genus $g > 2$. His proof is not effective, in the sense that the finite number of rational points obtained from his proof is too large to be computationally useful. On the other hand, the Chabauty-Coleman method does give an explicit upper bound (which is sometimes exact!) on the number of rational points on certain curves of genus $g > 2$, although it applies to a smaller class of curves. In this talk, we discuss the generalization of the Chabauty-Coleman method to search for rational points on certain higher-dimensional varieties; namely, we will discuss the case of symmetric powers of curves.

Scott Parsell, West Chester University

The Underlying Congruences in Waring's Problem

Let $\Gamma(k)$ denote the smallest integer s for which the congruence $x_1^k + \cdots + x_s^k \equiv a \pmod{m}$ has a solution for each residue class a and every modulus m. In contrast to the function $G(k)$ in the global version of Waring's problem, surprisingly little is known about the behavior of $\Gamma(k)$ in general. We report on recent computational work in this direction and describe the connection with finding small primes in arithmetic progressions.

Charles Samuels, Christopher Newport University

The Continuous Adeles on the Field of Algebraic Numbers

In 2009, Allcock and Vaaler defined a topology on the set Y of places of $\overline{\mathbb{Q}}$. While they applied this topology to study a certain Banach space, we use it for a different purpose – to create an analog of the adèles for $\overline{\mathbb{Q}}$ called the *continuous adèles*. The usual finiteness condition, which is enforced in the definition of the adèles of a number field, is replaced with a compactness condition and a continuity condition. When equipped with an appropriate topology, we observe the continuous adèles form a Hausdorff topological ring which contains the adèle ring of every number field as a topological subring.

Lola Thompson, Oberlin College

Divisor-sum Fibers

Let $s(\cdot)$ denote the sum-of-proper-divisors function, that is, $s(n) = \sum_{d|n,\ d<n} d$. Erdős–Granville–Pomerance–Spiro conjectured that, for any set $\mathcal{A}$ of asymptotic density zero, the preimage set $s^{-1}(\mathcal{A})$ also has density zero. We prove a weak form of this conjecture. In particular, we show that the EGPS conjecture holds for infinite sets with counting function $O(x^{\frac{1}{2}+\epsilon(x)})$. We also disprove a hypothesis from the same paper of EGPS by showing that for any positive numbers α and ϵ, there are integers n with arbitrarily many s-preimages lying between $\alpha(1-\epsilon)n$ and $\alpha(1+\epsilon)n$. This talk is based on joint work with Paul Pollack and Carl Pomerance.

Alexander Walker, Brown University

Long Arithmetic Progressions in Digit-Excluding Sets

What is the length of the longest arithmetic progression that excludes a base-b digit? In this talk, I'll provide an exact expression for this maximal length, as a function of the base. The solution introduces an arithmetic function of independent interest: $\rho(n)$, the largest integer less than n whose radical divides n. Techniques from sieve theory and integer linear programming may be used to estimate this arithmetic function and, consequently, the lengths of digit-excluding progressions. Joint work with Aled Walker.

John Webb, James Madison University

A Simple Plan for Calculating Bases of Modular Forms

We develop a new algorithm to compute a basis for $M_k(\Gamma_0(N))$, the space of weight k holomorphic modular forms on $\Gamma_0(N)$, in the case when the graded algebra of modular forms over $\Gamma_0(N)$ is generated at weight two. Our tests show that this algorithm significantly outperforms a commonly used algorithm which relies more heavily on modular symbols.

Cassie Williams, James Madison University

An exact product formula for abelian varieties of odd prime dimension

Consider f, the characteristic polynomial of Frobenius of an abelian variety of odd prime dimension over a finite field. We use f to define and compute an infinite product of local relative densities of matrices in $\mathrm{GSp}_{2g}(\mathbb{F}_\ell)$ with characteristic polynomial $f \mod \ell$. The product, as in Gekeler (2003) and Achter-Williams (2015), is closely related to a ratio of class numbers. Using work of Howe, we conjecture an application of our formula to the size of an isogeny class of principally polarized abelian varieties of odd prime dimension.