

**Math 490 - Spring 2016**  
**Midterm Practice**

*In mathematics you don't understand things. You just get used to them.*

— John von Neumann

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**Practice:**

(1) Find the ordinary generating function for the following sequences:

(a)  $a_n = 4$

(b)  $a_n = n + 1$

(c)  $a_n = 3^n$

(d)  $a_n = a_{n-1} + 2a_{n-2}$ ,  $a_0 = 1$ ,  $a_1 = 1$ .

(2) Find a generating function for the number of Dyck-Path-like walks that consist either of

- Up-Steps of slope 1, (length 1)
- Half-Up-Steps of slope 1/2 (length 2)
- Down-Steps of slope -1 (Length 1)

Hint: The sequence begins: 0,1,1,2,4,7... Draw pictures!

(3) For a fixed integer  $k \geq 0$ , find the exponential generating function for  $\left\{ \binom{n}{k} \right\}_{n=0}^{\infty}$

(4) Suppose  $f(n)$  is a function that satisfies  $n^2 = \sum_{d|n} f(d)$ . Use möbius inversion to find a

formula for  $f(n)$ , and use it to compute the values of  $f(n)$  for  $1 \leq n \leq 6$ .