Math 490 - Spring 2016
Midterm Practice

In mathematics you don't understand things. You just get used to them.

— John von Neumann

Practice:

(1) Find the ordinary generating function for the following sequences:
   (a) $a_n = 4$
   (b) $a_n = n + 1$
   (c) $a_n = 3^n$
   (d) $a_n = a_{n-1} + 2a_{n-2}$, $a_0 = 1, a_1 = 1$.

(2) Find a generating function for the number of Dyck-Path-like walks that consist either of
   - Up-Steps of slope 1, (length 1)
   - Half-Up-Steps of slope 1/2 (length 2)
   - Down-Steps of slope -1 (Length 1)

   Hint: The sequence begins: 0,1,1,2,4,7... Draw pictures!

(3) For a fixed integer $k \geq 0$, find the exponential generating function for $\left\{ \binom{n}{k} \right\}_{n=0}^{\infty}$

(4) Suppose $f(n)$ is a function that satisfies $n^2 = \sum_{d|n} f(d)$. Use möbius inversion to find a formula for $f(n)$, and use it to compute the values of $f(n)$ for $1 \leq n \leq 6$.