

Math 490 - Fall 2015

Homework 1

Due November 9, 2017

In mathematics you don't understand things. You just get used to them.

— John von Neumann

Turn in:

(1) Assume a and b are constants. Find the ordinary power series generating functions of each of the following sequences, in simple, closed form. In each case the sequence is defined for all $n \geq 0$.

(a) $a_n = an + b$

(b) $a_n = n^2$

(c) $a_n = n2^n$

(d) $a_n = \begin{cases} 1 & n \equiv 0 \pmod{2} \\ 0 & n \equiv 1 \pmod{2} \end{cases}$

(2) Find a generating function for the sequence defined by $a_{n+1} = 3a_n + 2$, $a_0 = 0$, and use this generating function to find a closed form for this sequence.

(3) Find a simple formula for each of the following (assuming $n \geq 1$).

(a) $\sum_{k=0}^n (-1)^k \binom{n}{k}$.

(b) $\sum_{k=0}^n \binom{4n}{4k}$. (Hint: $(1+i)^4 = (1-i)^4 = -4$.)