

## Math 465 - Spring 2021

### Homework 6

Due March 26th, 2021

*I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our "creations," are simply our notes of our observations. This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards, and I shall use the language which is natural to a man who holds it.*

— G. H. Hardy

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- (1) Suppose  $f(n) = \frac{n-1}{n}$  and  $g(n)$  is a function with the property that

$$f(n) = \sum_{d|n} g(d).$$

Compute  $g(24)$ .

- (2) Define  $\sigma_k(n) = \sum_{d|n} d^k$ . Note that  $\sigma_1(n) = \sigma(n)$  and  $\sigma_0(n) = d(n)$ . Prove that  $\sigma_k(n)$  is multiplicative for any value of  $k$ . (Hint: you may want to prove that  $f(n) = n^k$  is multiplicative.)
- (3) Prove that  $d(n) \leq 2\sqrt{n}$  holds for all positive integers  $n$  (Hint if  $a|n$  then at least one of  $a$  or  $\frac{n}{a}$  is at most  $\sqrt{n}$ .)
- (4) Exercises 6-2.1 and 6-4.11 from the book.