

Math 465 - Spring 2021

Homework 4

Due February 25th 2021

There are certain things whose number is unknown. If we count them by threes, we have two left over; by fives, we have three left over; and by sevens, two are left over. How many things are there?

— Sunzi Suanjing, 3rd century AD

- (1) Let $m, n > 1$ be (not necessarily coprime) integers. Prove that the two congruences $x \equiv a \pmod{m}$ and $x \equiv b \pmod{n}$ admit a simultaneous solution if and only if $\gcd(m, n)$ divides $a - b$.
- (2) Use the previous problem to show that there is no simultaneous solution to $x \equiv 5 \pmod{6}$ and $x \equiv 7 \pmod{15}$.
- (3) Use both Fermat's Little Theorem and the Chinese remainder theorem to show that if p and q are distinct primes then

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}.$$

- (4) 5-3.1(a,c) 5-3.4, 5-4.1 (From the textbook).