

When people tell you something's wrong or doesn't work for them, they are almost always right. When they tell you exactly what they think is wrong and how to fix it, they are almost always wrong.

— Neil Gaiman

- (1) Prove that $\mu(n)$ is multiplicative.
- (2) The Liouville λ -function is defined by $\lambda(1) = 1$ and $\lambda(p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}) = (-1)^{k_1+k_2+\cdots+k_r}$.
 - (a) Show that λ is totally multiplicative.
 - (b) Show that

$$\sum_{d|n} \lambda(d) = \begin{cases} 1 & n = m^2 \\ 0 & \text{otherwise} \end{cases}$$

- (3) Suppose $f(n)$ is multiplicative and $g(n) = \sum_{d|n} f(d)$. Prove that $g(n)$ is multiplicative. Hint: the goal is to show that $g(mn) = g(m)g(n)$. Foil out the expression $\left(\sum_{d|m} f(d)\right) \left(\sum_{e|n} f(e)\right)$.