Math 465 - Spring 2021 February 25th, 2021
Group Discussion 5

For every problem, there is one solution which is simple, neat, and wrong.

— H. L Mencken

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(1) Compute $\varphi(6)$, $\varphi(10)$, $\varphi(15)$, $\varphi(21)$.
(2) Fix p, q, two distinct primes. Consider the function $f : \mathbb{Z}_{pq}^* \to \mathbb{Z}_p^* \times \mathbb{Z}_q^*$ given by

$$f(k) := (k \bmod p,\ k \bmod q)\,.$$

  (a) Show that f is well defined.
  (b) Show that f is one-to-one.
  (c) Show that f is onto. (*Hint:* Chinese Remainder Theorem.)
  (d) Conclude that $\varphi(pq) = \varphi(p)\,\varphi(q)$.

(3) Do Tasks 8-14 in the Mobius file.