Math 565 - Fall 2019
Homework 6
Due October 28th, 2019

I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our "creations," are simply our notes of our observations. This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards, and I shall use the language which is natural to a man who holds it.

— G. H. Hardy

Problem 6.4-11 from the book and the following.

(1) Suppose $f(n) = \frac{n-1}{n+1}$ and $g(n)$ is a function with the property that

$$f(n) = \sum_{d|n} g(d).$$

Compute $g(24)$.

(2) For which prime $p < 20$ is there the most primitive roots? (Hint: don't actually find the primitive roots...)

(3) Using the information that 7 is a primitive root modulo 22, determine all integers modulo 22 that are primitive roots.

(4) (a) Determine the order of 2 modulo $2^n - 1$.
(b) Use this to prove that $\varphi(2^n - 1)$ is divisible by n.