

**Math 465 - Fall 2019**

**Homework 10**

Due December 11, 2019

*Mathematics is the queen of sciences and number theory is the queen of mathematics.*

— Carl Gauss

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(1) Problem 11-1.1.

(2) By imitating our proof in class (using Thue's Theorem) and the symbol  $\left(\frac{-2}{p}\right)$ , show that any prime  $p \equiv 1, 3 \pmod{8}$  can be written as

$$x^2 + 2y^2 = p$$

in integers,  $x$  and  $y$ .

(3) Show that

$$\lim_{N \rightarrow \infty} \frac{\sum_{n=0}^N r_3(n)}{N^{3/2}} = \frac{4\pi}{3}$$

where  $r_3(n)$  counts the number of ways to represent  $N = x^2 + y^2 + z^2$ . Hint: follow the method of proof of the result for  $r_2(n)$ , using a sphere and unit cubes.