3/01/2018 Class Notes

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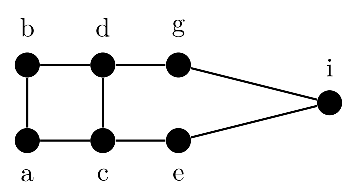
Prim's Algorithm:

- Pick any vertex V, then pick any edge with the lowest weight connected to vertex V.
- Repeat. Taking the edge with the lowest weight that connects the existing tree to a vertex not already connected to it.

How many spanning trees does a connected graph have?

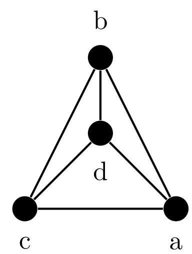
- Example:



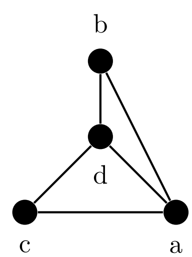
  - Let the edge from d to c be represented by e1
  - First count the trees in the graph including e1
    * There are 3 possible edges to remove on the left side of e1
    * There are 4 possible edges to remove on the right side of e1
    * This means that including e1, there are 12 total possible spanning trees $(4 * 3)$
  - Now count the trees in the graph not including e1
    * There are 7 possible edges to remove and removing any of them results in a spanning tree.
  - Therefore are 19 possible spanning trees total because $12 + 7 = 19$

How many spanning trees are there on n labelled vertices if any edge is allowed?

- Counting spanning subtrees of a complete graph of order n
- Lets try this with n = 4

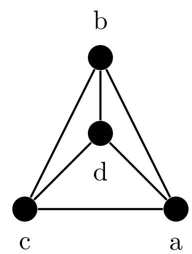


- Get rid of an edge



- Let the edge from vertex d to a be e2
  We want to count the number of spanning trees in our graph G including e2, as well as not including e2.
- Number of trees including e2:
  2 on left * 2 on right = 4 total
  Number of trees not including e2:
  4 total
  All together there are 8 in total.

- Now lets try it including the original edge we removed (the edge from vertex b to vertex c). Call it e1.



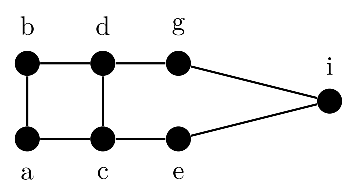
- Now we repeat the same process as above. In our new graph, if we include e2, we have 4 possible spanning trees. If we don't include our edge e2 we have another 4 possible spanning trees.
  If we multiply these together we get 16 total possible spanning trees.

- The equation to calculate the number of spanning subtrees for a complete graph is n^{n-2}

Matrix Tree Theorem:

- Given a connected graph G from the matrix A where:
  $A_{ij} = deg(V_i)$
  $A_{ij} = -1$ if V_iV_j exist within E(G), 0 if otherwise

- Example:



- $A = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$

- Then the number of spanning trees of our graph is equal to the determinant of the matrix $[A_{ij}]$ where this is any cofactor of the matrix obtained by removing row i and column j

- This gives us an easy way to computer the number of spanning trees even if the graph is really large.