# Class Notes

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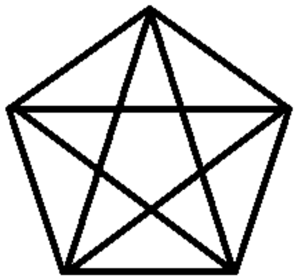
## 1 Homework Problem Review

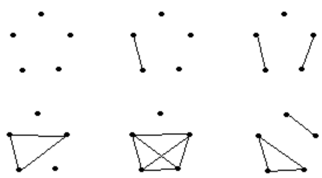
Question One: Draw all connected graphs of 5 vertices with the distance between any two distinct vertices is odd.



This is K5, the only graph with those properties

Proof: Since distance in this graph can't be more than 4 the only odd distance are 1 and 3. If 2 vertices are distance 3 then the path between them would contain two vertices of distance 2. Meaning the distance must be 1 between all points for the conditions to be met, and K5 is the only connected graph.

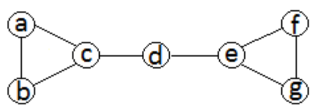
Question Two: Draw all disconnected graph with previous conditions.



These are only the six disconnected graphs above.

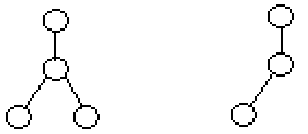
## 2 Bridges, Forest, and Trees

Definition 1: A **bridge** in a connected graph is an edge with the property that if the edge is removed from the graph the graph is no longer connected.



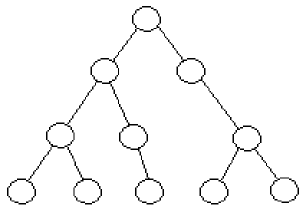

The edge c,d is a Bridge, but the edge f,g is not a Bridge.

Definition 2: A graph which has no cycles is called a **forest**.



The two components above form a Forest.

Definition 3: A **tree** is a connected forest.



The above image is a tree

# 3 Claims About Trees

## 3.1 Claim 1:

If G is a tree, then any edge in the graph in G is a bridge.

Proof: suppose G is a tree with atleast two vertices (otherwise G has no edges and follows trivially). Let e be an edge in G, with the vertices u and v. Assume e is not a bridge. Then there exist a cycle in G making G not a tree, Contradicting the assumptions that G is a tree. Meaning G must be a bridge.

## 3.2 Claim 2:

If G is a connected graph where every edge is a bridge the G is a tree.

Proof: Suppose G is a connected graph where every edge is a bridge and G is not a tree. For G to not be a tree G must contain a cycle, U1,U2,...,U1. You can remove any edge from the cycle and the graph is still connected. meaning the edges in the cycle are not bridges, a contradiction that every edge is a bridge. Meaning G is tree.