Notes 2-15

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Continuation of 2/13 Notes

Definition: An irregular graph is a graph in which no two distinct vertices have the same degree.

Example: Trivial Graph

Theorem: There do not exist any non-trivial irregular graphs.

Proof: If a graph has order n then all of its n vertices have degrees between (0 and n-1): 0,1,2,3,...,(n-1)

Suppose for contradiction that G is an irregular graph of order n. Thus G must have a vertex with every degree from 0 to n-1. So one vertex has degree 0 (connected to nothing) and one has degree n-1 (connected to everything)

This is a contradiction.

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1 Isomorphisms

We want a mathematical definition of what it is for two graphs to be the same

Definition: If G and H are two labelled graphs then

$$\varphi : V(G) \to V(H) \tag{1}$$

is an isomorphism (of Graphs).

If φ is a bijection and

$$u, v \in V(G) \quad \text{then} \quad u, v \in E(G) \quad \text{if and only if} \quad \varphi(u)\varphi(v) \in E(H) \tag{2}$$

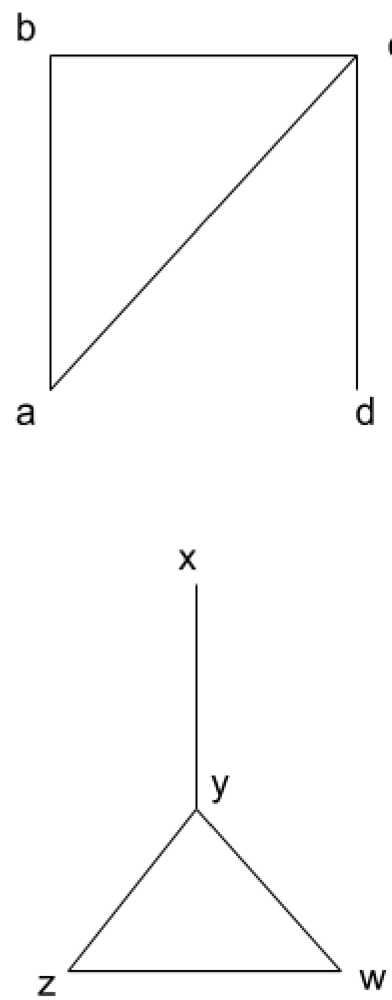


Figure 1: Two labelled graphs are isomorphic if there exists an isomorphism between them Two unlabelled graphs are isomorphic if there exists a way to assign labels to vertices in a way that makes them isomorphic.

2

In order for two graphs to be isomorphic they need to be the same order and size

If G and H are isomorphic then they have the same degree sequence

How do we tell if two graphs are isomorphic? What are the possible unlabelled graphs of small order?

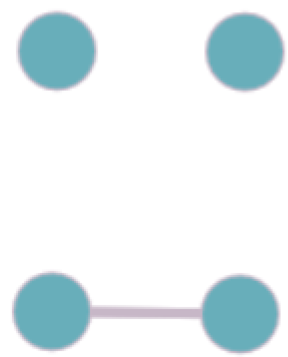


Figure 2: n=2

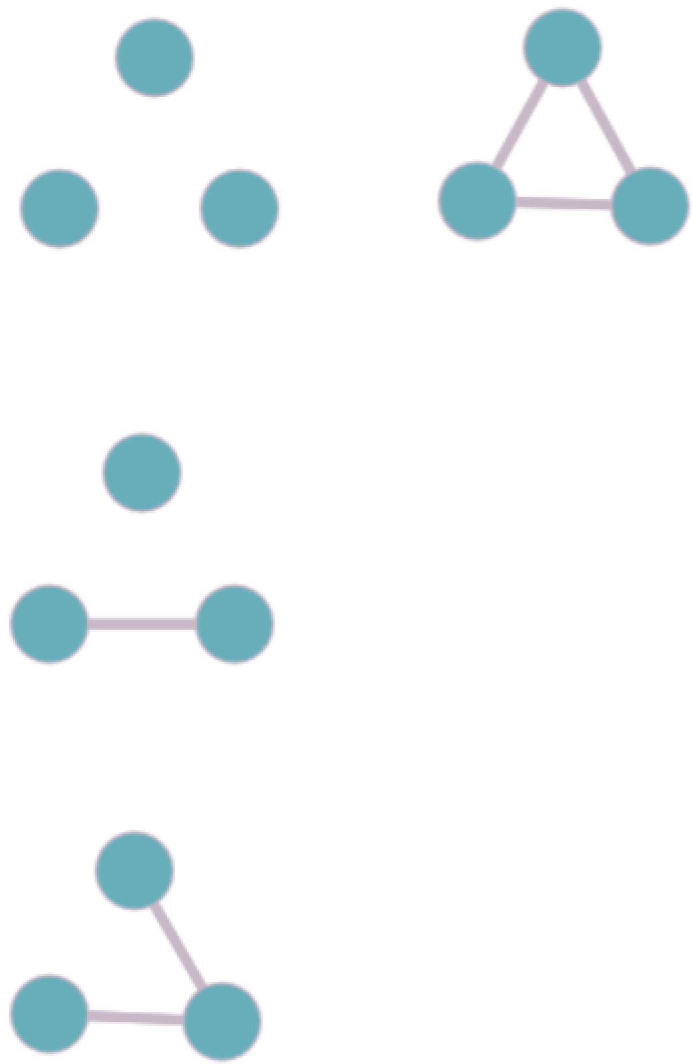


Figure 3: n=3

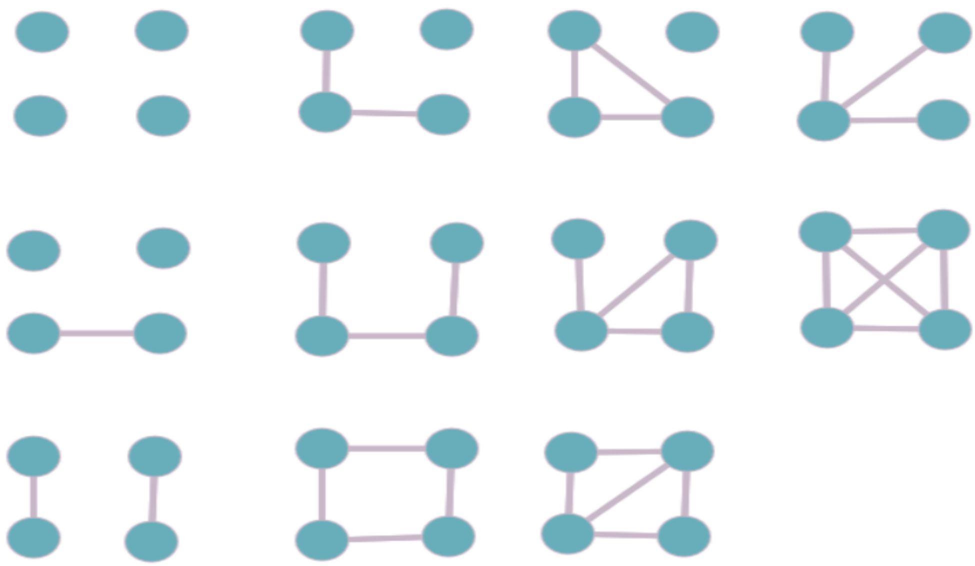


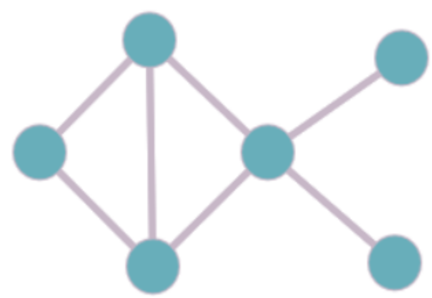
Figure 4: n=4

There is a relationship between the number of vertices n and the number of non isomorphic unlabelled graphs However we do not know a formula in terms of n for this sequence.

3

Theorem: G and H are isomorphic if $\overline{\mathrm{G}}$ is isomorphic to $\overline{\mathrm{H}}$

Write G $\cong$ H if G is isomorphic to H. -Determine whether the following graphs are isomorphic.



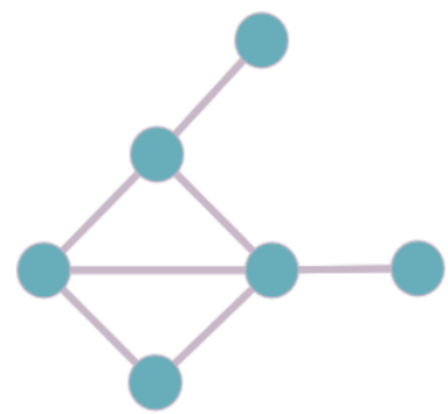


Figure 5: (Left): Dseq - 4,3,3,2,1,1

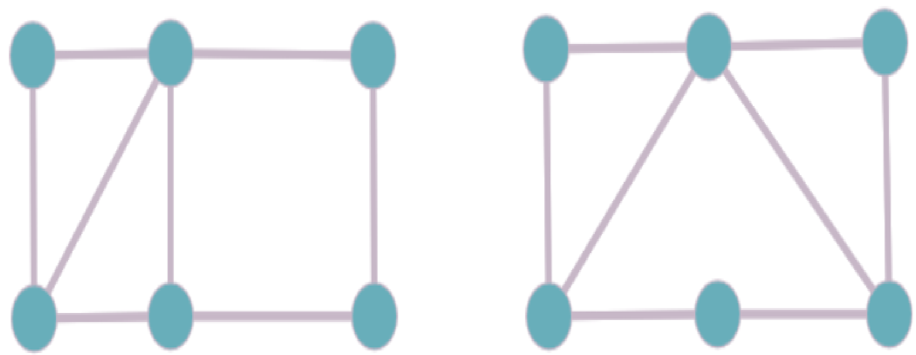
Figure 6: (Right): Dseq - 4,3,3,2,1,1

-If yes we can see this by writing down the isomorphism. -If not we need to demonstrate some property of one of the graphs that isnt preserved in the other.

Check $\rightarrow$ Order: 6, Size: 7, Degree Sequence: Same

In this case, distance between the vertices of degree 1 is not preserved.

Another way is to look at the neighbors of the unique vertices. In this case we examine the unique vertex of degree 4.



Note: No vertex of degree 2 has neighbors of 2 and 4 for the right graph.

Check: The distances between 2 points and the existence of cycles of different sizes.

Theorem: The property of being an isomorphic graph is an equivalence relation

Proof: Transitivity, Suppose F $\cong$ and G $\cong$ H Let φ:F→G be an isomorphism and ψ: G→H be an isomorphism

Then φ×ψ: F→H: Check that this is an isomorphism.

An isomorphism from G to itself is called an automorphism.

4

In K_n any permutation of the vertices is an automorphism. The set of automorphisms of a graph form a group under composition.

- Automorphism group of K_n is S_n
- Frudnts Theorem: For any finite group G. There exist a graph whose automorphsim group is G.