01/30/2018 Class Notes

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The following will be notes that were taken during the first lecture of class (01/30/2018). This lecture went over sections 1.1 1.2. This lecture consisted of multiple definitions to prepare students for the remainder of the class lectures.

0.1 Set

A set S is a collection of objects which can appear in the set at most once.

Ex: $S = \{a, b, c, d\}$

Not an Ex: $S = \{a, a, b\}$

Notation:

Write $a \in S$ **if a is an element of S.**

$z \notin S$ **if z is not an element of S.**

0.1.1 Subset

$A \subset B$ **If A is a subset of B.**

Ex: $\{a, b\} \subset \{a, b, c\}$

Not an Ex: $S = \{a, a, b\}$

$A \subseteq B$ **If A is a subset of B that might be the entire set.**

So A could equal B.

0.2 Graph

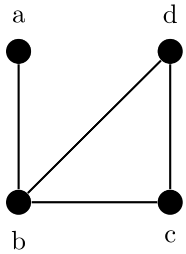
A graph G is an ordered pair $\{V, E\}$ **where V is a finite set of vertices and E is a set of pairs of vertices called edges**

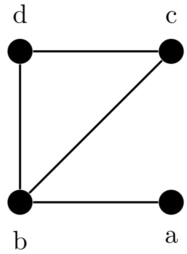
Example: $G = \{\{a,b,c,d\}, \{\{a,b\},\{b,c\},\{c,d\},\{d,b\}\}\}$

Vertices $= \{a,b,c,d\}$

Edges $= \{\{a,b\},\{b,c\},\{c,d\},\{d,b\}\}$

The following two graphs below are the same graph.





Note: When creating the graphs, the 'dots' can be put anywhere. It does not matter how you draw the graph, as long as the edges connect together the same way.

0.2.1 Edges

Notation for edges UV **for the edge** $\{u,v\}$

$$UV = \{u,v\}$$

We can say that U and V are adjacent, neighbors or joined, are all words meaning that there is an edge from u to v.

V(G) = Set of vertices of G

E(G) = Set of edges of G

Example:

$$V(G) = \{a,b,c,d\}$$

$$E(G) = \{ab,bc,cd,db\}$$

0.3 Order

The Order of a graph is the number of vertices in the graph.

0.4 Size

The Size of a graph is the number of edges in the graph.

0.5 Trivial Graph

The trivial graph is the unique graph with one vertex.

Sometimes we don't care about the names of the vertices and we only care about the shape.

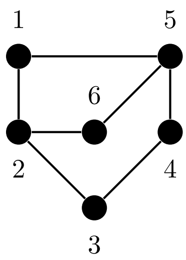
Call a graph an unlabeled graph if we only care about the structure of the graph and not the vertex labels.

A labeled graph is when each vertex has a unique name.

$G = \{\{i \mid 1 \leq i \leq 6\}, \{i, j\} \mid i - j = n^2\}\}$

In English: Take all pairs $\{i, j\}$ where i and j are valid vertices and include them in the edge set IF their difference is a square.

We do not care what square they make.

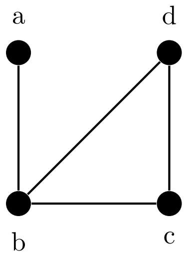


0.6 Subgraph

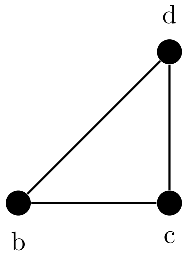
Def: Say H is a subgraph of G and write $H \subseteq G$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$

Example:

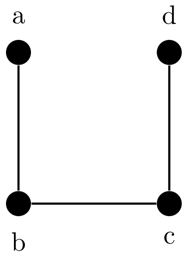
G =



H1 =



H2 =



H3 =

H1, H2, and H3 are all subgraphs of G.

0.7 Induced Subgraph

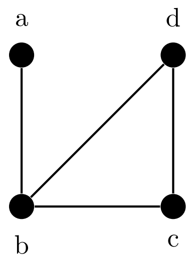
Def: Say H is an induced subgraph of G if $V(H) \subset V(G)$ **and for any two** $U, V \in V(H)$

$UV \in E(H)$ **if** $UV \in E(G)$

We can think of an induced subgraph as being a subset of the verticies with all of the edges still included if their vertices are.

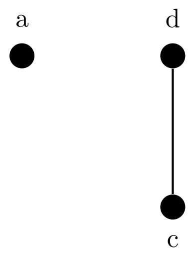
Say that H is the <u>**subgraph induced by S**</u> **if** $S \subset V(G)$ **and H is the graph with vertex set S that is an induced subgraph of G.**

G =



Subgraph induced by $\{a, c, d\}$ **we get:**

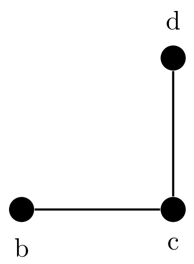
H =



Def: H is an edge-induced-subgraph of G if $E(H) \subset E(G)$ **and the vertex set of H consists of all vertices with at least one edge connected.**

Edge induced subgraph of the set $\{bc, cd\}$ **. This is a subset of the edges of G. The edge induced subgraph is:**

H =

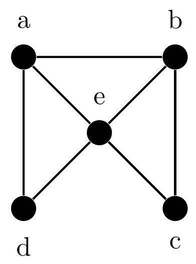


NOTATION:

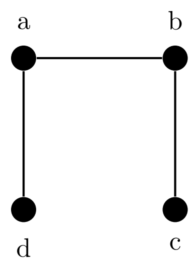
$G - V$ **is the subgraph induced by the vertex set** $V(G)\ \{V\}$ **.**

This is the graph with all of the vertices in G besides V and all of the edges except the ones that were connected to V.

G =



G - e =



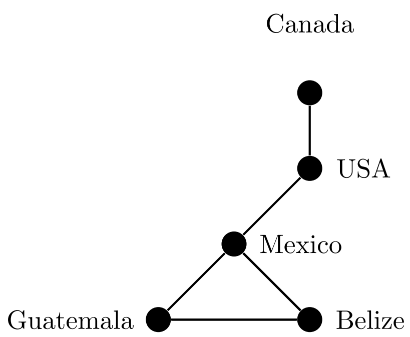
1 Moving on a Graph

How can we move around on a graph?

1.1 Walk

A Walk on a graph G is a sequence of vertices $(V_1, V_2, V_3......V_e)$

where any two consecutive vertices in the walk are connected by an edge.



A walk about be

(USA, Canada, USA, Mexico, Guatemala)

1.2 Trail

A Trail is a walk in which no edge occurs multiple times in the walk.

$$Example : (Mexico, Guatermala, Belize)$$

$$Example : (Mexico, Guatermala, Belize, Mexico, USA)$$

Both examples above are trails because we didn't go on multiple edges. We only went over multiple vertices.

1.3 Path

A Path is a walk where no vertex appears multiple times.

Note: Every path is also a trail.