

Math 451 - Spring 2018

Homework 8

Due April 24th, 2018

A good programmer is someone who looks both ways before crossing a one-way street.

— Doug Linder

Turn in:

- (1) Let G be an n -vertex digraph with no cycles. Prove that the vertices of G can be ordered as v_1, \dots, v_n so that if $v_i v_j \in E(G)$, then $i < j$.
- (2) Prove that there is an n -vertex tournament with in-degree equal to out-degree at every vertex if and only if n is odd.
- (3) A tournament T is *irreducible* if it is impossible to split the vertices of T into two non-empty disjoint sets V_1 and V_2 such that each arc joining a vertex of V_1 with V_2 is directed from V_1 to V_2 . Give an example of an irreducible tournament. Prove that a tournament T is strong if and only if it is irreducible.
- (4) A *king* in a directed graph is a vertex from which one can reach any other vertex in at most two steps. We proved in class that in a tournament the vertex of maximal outdegree is a king.
 - (a) Draw a strong tournament on 5 vertices with at least one vertex that is not a king.
 - (b) For every odd n , explain how to construct an n -vertex tournament in which *every* vertex is a king.
 - (c) Does there exist a tournament where every vertex is a king with 4 vertices?