

Math 451 - Spring 2018

Homework 7

Due April 12th, 2018

In addition to that branch of geometry which is concerned with magnitudes, and which has always received the greatest attention, there is another branch, previously almost unknown, which Leibniz first mentioned, calling it the geometry of position. This branch is concerned only with the determination of position and its properties; it does not involve measurements, nor calculations made with them. It has not yet been satisfactorily determined what kind of problems are relevant to this geometry of position, or what methods should be used in solving them.

— Euler, in his paper on the Königsberg bridge problem

Turn in:

- (1) Draw an Eulerian graph that is not Hamiltonian that has even order and odd size.
- (2) Prove that any regular graph of order $n \geq 3$ with at least $\frac{n^2}{4}$ edges is Hamiltonian.
- (3) Prove that $\overline{C_n}$ is Hamiltonian if $n \geq 5$.
- (4) The n dimensional hypercube Q_n is the graph with vertices

$$V = \{(a_1, a_2, \dots, a_n) \mid a_i \in \{0, 1\}\},$$

and an edge between (a_1, \dots, a_n) and (b_1, \dots, b_n) if

$$\#\{1 \leq i \leq n \mid a_i = b_i\} = n - 1.$$

- (a) How many vertices does Q_n have?
 - (b) What are the degrees of the vertices?
 - (c) Why is Q_n called a hypercube?
 - (d) Show that for $n \geq 2$, Q_n has a closed Hamiltonian path.
- (5) Consider the graph G_n defined in class with vertices $V = \{v_1, v_2, \dots, v_n\}$ and edges $E = \{v_i v_j \mid i + j = k^2\}$.
 - (a) Draw G_{14} .
 - (b) Show that G_n is not Hamiltonian for $2 < n \leq 30$.
 - (c) (Extra Credit) Show that G_{31} is not Hamiltonian.