Math 451 - Spring 2018
Homework 6
Due March 29nd, 2018

A good stock of examples, as large as possible, is indispensable for a thorough understanding of any concept, and when I want to learn something new, I make it my first job to build one.

— Paul Halmos

Turn in:

(1) Give an example of a connected graph G with vertex v for each of the following:
   (a) v is on a cycle and v is a cut-vertex.
   (b) v is on a cycle and v is *not* a cut-vertex.
   (c) v is *not* on a cycle and v is a cut-vertex.
   (d) v is *not* on a cycle and v is *not* a cut-vertex.

   Conclude that there is no relationship between the idea of cut-vertices and cycles.
(2) Draw a graph G with $\kappa(G) = 2$, $\lambda(G) = 4$. Indicate the minimum vertex cut and minimum edge cut.
(3) Prove that a 3-regular graph has a cut vertex if and only if it has a bridge.
(4) Prove that if G is a graph of order $n \geq 3$ such that for every pair of non-adjacent vertices x, y we have $\deg x + \deg y \geq n$, then show G is nonseparable.
(5) A **chorded cycle**, in a graph, is a cycle C with an edge that joins two *nonconsecutive* vertices on C. Prove that every graph with $\kappa(G) \geq 3$ contains a chorded cycle.