Math 451 - Spring 2018
Homework 2
Due February 13, 2018

Different branches of mathematics require different aptitudes. [Problem Solvers or Theory Builders] In some, such as algebraic number theory, or geometry, it seems . . . to be important for many reasons to build up a considerable expertise and knowledge of the work that other mathematicians are doing, as progress is often the result of clever combinations of a wide range of existing results. Moreover, if one selects a problem, works on it in isolation for a few years and finally solves it, there is a danger, unless the problem is very famous, that it will no longer be regarded as all that significant.

At the other end of the spectrum is, for example, graph theory, where the basic object, a graph, can be immediately comprehended. One will not get anywhere in graph theory by sitting in an armchair and trying to understand graphs better. Neither is it particularly necessary to read much of the literature before tackling a problem: it is of course helpful to be aware of some of the most important techniques, but the interesting problems tend to be open precisely because the established techniques cannot easily be applied.

— W. T. Gowers, The two cultures of mathematics

Turn in:

(1) Go to `http://planarity.net/game.html`. (You will need a computer that has flash.) The goal is to drag the vertices around until there are no crossing edges. Play the first few levels, and take a screenshot of the game once you complete level 5. Submit this screenshot. (The screenshot should show the "Completed level 5" screen, with the completed graph with no crossings behind it.) Bonus points if your screenshot has the highest number of points in the class.

(2) (a) Draw all the unlabelled, connected graphs of order 5 in which the distance between any two distinct vertices is odd. Explain why you know you have drawn all of them.
(b) Draw all disconnected unlabelled graphs of order 5 with the property that the distance between any two distinct connected vertices is odd. (Note: if vertices are not connected, the distance between them is undefined.) (No justification is necessary for this part, just draw them all.)

(3) Let G be a bipartite graph of order at least 5. Prove that $\overline{G}$ is not bipartite.

(4) Give an example of each or explain why no such graph exists.
(a) A graph of order 7 whose vertices have degrees $1, 2, 2, 2, 3, 3, 4$.
(b) A graph of order 7 whose vertices have degrees $2, 2, 2, 3, 3, 4, 7$.
(c) A graph of order 7 whose vertices have degrees $1, 1, 1, 1, 1, 2, 5$
(d) A graph of order 6 whose vertices have degrees $1, 5, 5, 5, 5, 5$.

(5) (a) Prove that if G is a graph of order n such that every vertex has degree at least $(n - 2)/3$ then G has at most 2 components.
(b) Show that this bound is sharp.

Practice (Don't Turn in): Exercise 1.12