Math 451 - Spring 2018
Homework 1
Due February 6, 2018

There is no problem in all mathematics that cannot be solved by direct counting.

— Ernst Mach

Turn in:

(1) Consider the graph $G = (V, E)$ where $V = \{a, b, c, d, e\}$ and $E = \{ab, ac, bc, bd, de, ec\}$.
  (a) Draw the graph defined by G.
  (b) What is the size and order of G?
  (c) It is standard notation in graph theory to let $\delta(G)$ be the minimum degree and $\Delta(G)$ be the maximum degree of any graph G. What is $\delta(G)$ and $\Delta(G)$ in this example?
  (d) Draw the picture of $\overline{G}$.

(2) Give a formula for the number of edges in K_n.

(3) Let G be a graph of size 10. Assume it only has vertices of degree 1, 2 and 4. If we know it has 3 vertices of degree 2, and 2 vertices of degree 4, then how many vertices of degree 1 must G have?

(4) Prove that in any graph G there must be an even number of vertices with odd degree.

(5) Let $S \subset V$ be a subset of vertices in a graph G. Show that S is a clique in G if and only if S is an independent set in $\overline{G}$.

(6) Let $G = (V, E)$ be a graph. We say a graph $H = (V', E')$ is a **subgraph** of G and write $H \subseteq G$ provided that $V' \subseteq V$ and $E' \subseteq E$.
  Assume that H is a subgraph of G. Is it possible for:
  (a) $\Delta(G) < \Delta(H)$?
  (b) $\delta(G) < \delta(H)$?
  Be sure to justify your answer.

(7) Let $H \subseteq G$. We say H is an induced subgraph of G if for every $u, v \in V(H)$, then uv is an edge of H if and only if uv is an edge of G. Find graphs H and G such that $H \subseteq G$ but is not an induced subgraph of G.