Math 378 - Fall 2024
Homework 2
Due September 24, 2024

It is difficult to find a definition of combinatorics that is both concise and complete, unless we are satisfied with the statement "Combinatorics is what combinatorialists do.

— W.T. Tutte

Turn in:

(1) Let $\sigma = 6732415 = (1,6)(2,7,5,4)(3)$ and $\tau = 2345671$.
   (a) Compute $D(\sigma)$, $D(\tau)$, $I(\tau)$ and $I(\sigma)$.
   (b) Using that $T(\sigma) = 4$ and $T(\tau) = 6$, find a minimal set of transpositions representing each permutation.
(2) Suppose n is even. Give a formula for the number of permutations of n whose longest cycle has length $n/2$. (Note: it could be a tie!)
(3) Prove the following identity:
$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

   Hint: Use a combinatorial proof. Let S be a set of size n with three distinguished elements a, b, and c. Count the number of k element subsets of n That contain at least one of a, b or c in two different ways.