MATH 374 Notes

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1 Continuation of April 27 Notes

y(x) = $c_0 + c_1x + c_2x^2 + c_3x^3 + ...$
$= c_0 + c_1x + 0x^2 - c_0/6x^3 - c_1/12x^4 + 0x^5 + c_0/180x^6 + c_1/504x^7 + 0x^8 + ...$
$= c_o[1 - 1/6x^3 + 1/180x^6 + 1/12960x^9 + ...]$
$+ c_1[x - 1/27x^4 + 1/504x^7 - 1/45360x^10 + ...]$
Finding solutions
$y_1(x) = \sum_{k=0}^{\infty} 1*4*7...(3k-2)/(3k)!(-1)^k x^{3k}$
$= \sum_{k=0}^{\infty}\sum_{i=1}^{k}(3k-2)/(3k)!(-1)^k x^k$
$y_2(x) = \sum_{k=0}^{\infty} 2*5*8...(k-1)/(3k+1)!(-1)^k x^{3k+1}$
$y_2(x) = \sum_{k=0}^{\infty}\sum_{i=1}^{k}(3k-1)/(3k+1)!(-1)^k x^{3k+1}$

In many cases we won't write down the full series solution just solve for the first several terms of a series solution

These partial sums should be good approximates to the solution near the center point.

2 Example 1

Find the first 6 terms of a series solution to
(x-1)y" - xy' + y = 0
satisfying y(0) = -2 and y'(0) = 6
Centered at $x_o = 0$
Does this equation have singular points?
y" - x(x-1)$^{-1}y' + 1(x-1)^{-1}y = 0$
So we'll find a series solution around $x_o = 0 which converges on the interval (-1, 1)$
y(x) = $\sum_{n=0}^{\infty} x_n x^n$
y'(x) = $\sum_{n=1}^{\infty} nc_n x^{n-1}$
y"(x) = $\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}$
xy" - y" - xy' + y = 0
xy" = $\sum_{n=2}^{\infty} n(n-1)c_n x^{n-1}$
y" = $\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}$
xy' = $\sum_{n=1}^{\infty} nc_n x^n$
y = $\sum_{n=0}^{\infty} c_n x^n$

Therefore $c_o - 2c_2 + \sum_{n=2}^{\infty}[(k+1)kc_{k+1} - (k+2)(k+1)c_{k+2} - kc_{kck}]x^k$
$= c_o - 2c_2 + \sum_{n=2}^{\infty}[(k+1)kc_{k+1} - (k+2)(k+1)c_{k+2} - c_k(k-1)]x^k = 0$
$c_o - 2c_2 = 0 => c_2 = c_o/2$
$k(k+1)c_{k+1} - (k+2)(k+1)c_{k+2} - (k-1)c_k = 0$
$(k+2)(k+1)c_{k+2} = k(k+1)c_{k+1} - (k-1)c_k$
$c_{k+2} = k(k+1)c_{k+1} - (k-1)c_k$
y(0) = -2 =¿ c_o
y'(0) = 6 =¿ $c_1 = 6$
$c_2 = c_0/2 = -1$
$c_3 = c_{1+2} = -1/3$
$c_4 = -1/12$
$c_5 = -1/60$
$c_6 = -1/360$
y(x) = -2 + 6x - $x^2 - x^3/3 - x^4/12 - x^5/60 - x^6/360 + ...$

3 Example 2

We can do this when the coefficients aren't just polynomials too.

y" + sin(x)y = 0
Find the first 3 terms of a solution around 0 to y(0) = 1, y'(0) = 2
To solve this we need to use the Taylor Series for sin(x) around $x_o = 0$
sin(x) = x - $x^3/3! + x^5/5! + ...$
y(x) = $c_0 + c_1x + c_2x^2 + c_3x^3$
y'(x) = $c_1 + 2c_2x + 3c_3x^2 + ...$
y"(x) = $2c_2 + 6c_3x + ...$
Therefore:
$(2c_2 + 6c_3x) + sin(x)(c_0 + c_1 + ...) = 0$
$(2c_2 + 6c_3x + ...) + (x - x^3 + ...)(c_0 + c_1 + c_2x^2 + ...) = 0$
$(2c_2 + 6c_3x + ...) + (c_0x + c_1x^2 + c_2x^3 + ... - x^3c_0/3 - c_1/3!x^4 + ...)$
$2c_2 + (6c_3 + c_0)x + (12c_4 + c_1)x^2 + ... = 0$
Each coefficient is 0
Therefore:
$2c_2 = 0$
$6c_3 + c_0 = 0$
$12c^4 + c_1 = 0$
$c_0 = 1$
$c_1 = 2$
$c_2 = 0$
$c_3 = -1/6$
$c_4 = -1/6$
.
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