Initial Value Problem (IVP)

Let R be a rectangular region of the X,Y plane
$a < x < b$
$c < y < d$
and (x_0, y_0) a point in the interior of R

If F(x,y) and $\frac{df}{dy}$ are continuous on R then there exists an internal I containing x_0 on which the IVP $\frac{df}{dy} = F(x, y)$ has a unique solution.

Recall an example of IVP without a unique solution
$\frac{dy}{dx} = xy^{\frac{1}{2}}$, $y(0) = 0$
$y = \frac{x^4}{16}$
$y = 0$

So why didnt the theorem apply in this case?

$(x_0, y_0) = (0, 0)$
$F(x, y) = xy^{\frac{1}{2}}$
$\frac{\partial f}{\partial y} = \frac{1}{2}xy^{-\frac{1}{2}} = \frac{x}{2y^{\frac{1}{2}}}$ which is not defined at (0,0)

DEs occur a lot in various real world modeling scenarios such as:

Population Dynamics

- P = the number of individuals alive at some given time(t)
- How does the population grow over time?
- How fast is the population changing ($\frac{dP}{dt}$)
- First approximation: $\frac{dP}{dt} = kP$, where k is some constant accounting for all environmental effects

Radioactive Decay

- $ln(t)$ = the mass of radioactive material at some time(t)
- How much radioactive material is left at some future time?
- the rate of decay, or half life, will be $\frac{dm}{dt}$
- $\frac{dm}{dt} = km$, where this time k depends on the isotope

Temperature

- An object with temperature T(t) is immersed in a fixed ambient temperature T_a
- What is the temperature of the object over some time T
- How fast is the temperature changing = $\frac{dT}{dt}$
- $\frac{dT}{dt} = k(T_A - T)$, which is Newton's law of heating/cooling

F=ma

- $Force = Mass * Acceleration$
- $F = mg$, where g is the gravitational constant

LRC Circuits

- Take current flowing through a wire that has a resistor(R), capacitor(C), and Inductor(L)
- Kerckhoff's Law:
  - Total drop in voltage is equal to the sum of the drops at each component
- Q = amount of charge
- I = current in the wire = $\frac{dQ}{dt}$ = change in current over time
- Change in voltage for a capacitor is: $\frac{1}{C}Q$
- Change in voltage for a resistor is: $RI = R\frac{dQ}{dt}$
- Change in voltage for a inductor is: $L\frac{dI}{dt} = L\frac{d^2Q}{dt^2}$
- $E(t) = \frac{1}{C}Q + R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2}$

Direction Fields

- If F(x,y,y') is a first order DE then the direction field for F is a plot with short line segments denoting the slope y' at each (x,y)
- Solutions to the DE will be tangent to each line segment in the direction field