

1. Consider the differential equation $xy'' - 2y' + \left(x + \frac{2}{x}\right)y = x^2$.
 - (a) Verify that $y_1 = x \sin x$ and $y_2 = x \cos x$ are solutions to the corresponding homogeneous equation.
 - (b) Use variation of parameters to find a particular solution to this equation.
 - (c) What is the general solution?
2. Solve $y''' - 3y' + 2y = \sin(2x + 1)$ using variation of parameters.
3. Solve $x^2y'' + 7xy' + 9y = 0$ with the initial conditions $y(1) = 2$, $y'(1) = 3$.
4. Consider $x^2y'' - 3xy' - 5y = \ln(x) + x^2$.
 - (a) Use the techniques of Cauchy-Euler equations to solve the homogeneous equation.
 - (b) Use variation of parameters to find a particular equation for $f(x) = \ln x$.
 - (c) Use variation of parameters to find a particular equation for $f(x) = x^2$.
 - (d) Find the general solution.