MATH 369 Fall 2025 - Class Notes

11/23/2025

Scribe: Muswe P

No official class this day but the group assignment for those who came to class was to create a diagram (Venn Diagram, or bulleted list or otherwise) on the board comparing and contrasting group homomorphisms with ring homomorphisms (along with kernels/fundamental homomorphism theorems etc.).

Rings

- Rings have two operations.
- The kernel has elements that go to 0.
- $\phi(a+b) = \phi(a) + \phi(b)$
- $\phi(a \cdot b) = \phi(a) \cdot \phi(b)$
- Sends $\phi(1) = 1$ by proof $\phi(0) = 0$ by assumption
- Not every element is invertible under multiplication

What They Have in Common

- There exists an identity element
- Kernel closed under operations
- $\ker(\phi)$ = every element that maps to the identity
- Have images
- Have preimages
- Associative
- Nonempty
- Bijective $\Leftrightarrow$ Isomorphism
- Let $\phi : A \to B$
  $A/\ker(\phi) = \mathrm{Im}(\phi)$ (First Isomorphism Theorem, Fundamental Homomorphism Theorem for rings)
- Both define an equivalence relation: $a \sim b$ if $\phi(a) = \phi(b)$
- $\phi(-a) = -\phi(a)$
- ϕ is an isomorphism, kernel is trivial Image is entire codomain

Groups

- Groups have one operation
- All elements have inverses under the operation
- $\phi(e) = e$