# Notes1

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For the purpose of these notes, suppose $f(x) : A \to B$ is a homomorphism

## 1 Group Homomorphisms:

- 1 Operation
- Every element has an inverse under the operation
- $f(e) = e$

## 2 Ring Homomorphisms

- Every element is sent to 0
- Not every function that preserves the operations from one ring to another is a homomorphism
- We assume homomorphisms send 1 to 1 by an axiom

## 3 Both

- The image has an identity
- The kernal forms a normal subgroup/ideal (normal subring)
- The kernal is every element that a homomorphism sends to the identity
- Like all functions, they have images and preimages
- Associative
- Non-empty domain and codomain
- Bijective $\to$ Isomorphism
- $A/ker(f)\tilde{=}Im(f)$

- Define an equivalence relation $\sim_f$ where $a \sim_f b$ means $f(x) = f(-x)$
- If the kernal is trivial, the homomorphism is an isomorphism $Im(f) =$ the codomain of f