Math 315 - Fall 2017
Homework 7
Due November 15, 2017
In mathematics you don't understand things. You just get used to them.

— John Von Neumann

Turn in one of the following: (You can turn in more for extra credit.)

(1) Let $B(x)$ be the generating function for the collection of all lattice paths with Up/Down steps which stay above the x-axis and have no peaks of height 1. (i.e., in a ballot sequence, a point where Alice and Bob are tied, after which Alice gets a vote followed by a vote for Bob.) For example, paths such as

are **NOT** permitted. Find the generating function $B(x)$ using the same ideas as we looked at in class.

(2) Find a generating function for sequences of integers $\{a_1, a_2, \ldots a_n$ where $1 \leq a_1 \leq a_2 \leq \cdots \leq a_n$ and $a_i \leq i$. Note: the empty sequence is considered such a sequence!

(3) Let $a_0 = 0$ and define $a_n = 2n \cdot a_{n-1} + n!$ for $n \geq 1$. Find an explicit formula for a_n by using the EGF for the sequence $a_0, a_1, a_2, \ldots$. (Hint: Find a functional equation involving the EGF for a_n.)

(4) A permutation π of $[n]$ is said to be an **involution** if its cycle decomposition consists of only 1- or 2-cycles. Let $\mathcal{I}$ be the collection of all involutions (for all $n \geq 0$.) Find the EGF for $\mathcal{I}$.