Math 315 - Fall 2017
Homework 6
Due November 8, 2017

Let us subjugate a collection of objects taking into account their qualities and differences each from another, then we are lead, in our mathematical perspective, to the study of integers and their connecting operations, that is, we are lead to Number Theory.
If we, however, disregard the qualities of each individual object and only account for the difference between two objects insofar that they are different, then we are lead to investigations which are concerned with the position, the order, the choosing of these objects. This branch of mathematics is called Combinatorics.

— Eugen Netto

Turn in:

(1) (Exercise 26, Chapter 8) Let $b_0 = 1$ and for $n \geq 1$ let b_n satsify the following recurrence relation:

$$b_{n+1} = 3b_n + 2^n$$

(a) Find the generating function for the numbers b_n.
(b) Find a closed formula for b_n.

(2) (Exercise 27, Chapter 8) A certain kind of insect population multiplies so that at the end of each year, its size is the double of its size a year before, plus 1000 more insects. Assuming that originally we released 50 insects, how many of them will we have at the end of the nth year?

(3) Suppose we are creating fruit baskets containing apples, oranges, bananas, pears, and peaches with the following conditions:
- Each basket contains an even number of apples.
- The number of oranges in each basket is a multiple of three.
- Each basket contains at most 2 bananas, at most 1 pear, and at most 1 peach.

Let c_n be the number of such baskets that contain n pieces of fruit.
(a) Find the generating function for c_n.
(b) Use the generating function to find a formula for c_n.

(4) A permutation is indecomposable if it cannot be cut into two parts so that everything before the cut is smaller than everything after the cut. For example 3142 is indecomposable, but 2143 is not as you can cut it after the first two elements. Let f_n be the number of indecomposable permutations of length n and set $f_0 = 0$. Find the ordinary generating function $F(x) = \sum_{n=0}^{\infty} f_n x^n$ in terms of the generating function $G(x) = \sum_{n=0}^{\infty} n!x^n$, the generating function of all permutations.

(5) Find a generating function for the number of ways to make change for n cents using only pennies, nickles, dimes and quarters. Use this generating function to find the number of ways to make change for $1.99.