Math 315 - Fall 2017
Homework 5
Due October 30, 2017

A generating function is a clothesline on which we hang up a sequence of numbers for display.
— Herbert Wilf

Turn in:

(1) (a) Determine the number of positive integers $k \leq 15{,}000$ that are divisible by 3, 4, 5, or 10.

(b) Determine the number of positive integers $k \leq 15{,}000$ that are neither perfect squares nor perfect cubes.

(2) (a) Consider the following function:
$$f(x) = \frac{-1}{\sqrt{1 - x^2}}$$
Use Newton's Binomial Theorem to find the first four nonzero terms of the power series for $f(x)$.

(b) Take the integral of your answer to part (a) to find the first four nonzero terms of the power series for $g(x) = \cos^{-1} x$.

(3) (a) Consider the following function:
$$f(x) = \frac{x}{(1 - x)^2}$$
Find the power series for $f(x)$.

(b) Use your answer to part (a) to find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

(c) Find a function $g(x)$ that has power series $\sum_{k=1}^{\infty} k(k - 1)x^k$.

(d) Use your answer to part (c) to find the sum of the series $\sum_{n=1}^{\infty} \frac{n(n - 1)}{2^n}$.

(4) (Exercise 24, Chapter 8) Let $a_0 = 1$ and $a_1 = 4$ and for $n \geq 2$ let a_n satsify the following recurrence relation:
$$a_{n+1} = 8a_n - 16a_{n-1}$$
(a) Find the generating function for the numbers a_n.
(b) Find a closed formula for a_n.