Math 315 - Fall 2017
Homework 4
Due October 18, 2017

Last time, I asked: "What does mathematics mean to you? And some people answered: The manipulation of numbers, the manipulation of structures." And if I had asked what music means to you, would you have answered: "The manipulation of notes?"

— Serge Lang

Turn in:

(1) Find the number of permutations of length 6 who's square is the identity permutation. (What cycle types result in the identity when squared?)

(2) Let $\pi = p_1 p_2 \cdots p_n$ be a permutation of n (in one line notation). An inversion of π is a pair $\{p_i, p_j\}$ so that $i < j$ but $p_i > p_j$. Let us call a permutation *odd* (respectively *even*) of it has an odd (respectively even) number of inversions. Prove that any permutation of length n consisting entirely of one cycle is even if n is odd and odd if n is even.

(3) Let $a(n, k)$ be the number of permutations of length n with k cycles in which the entries 1 and 2 are in the same cycle. Prove that for $n \geq 2$:

$$\sum_{k=1}^{n} a(n, k) x^k = x(x + 2)(x + 3) \cdots (x + n - 1)$$

(4) A group of n tourists arrive at a restaurant. They sit down around an unspecified number of tables, leaving no table empty. Then each table orders one of the r possible drinks. Prove that the number of ways this can happen is

$$r^{(n)} = r(r + 1)(r + 2) \cdots (r + n - 1)$$

(5) Let $n \geq 6$ be even. How many permutations $p_1 p_2 \cdots p_n$ are there in which at least one of p_1, p_2 or p_3 is even?