Math 315 - Fall 2017
Homework 3
Due October 2, 2017

Like number theory before the 19th century, combinatorics before the 20th century was thought to be an elementary topic without much unity or depth. We now realize that, like number theory, combinatorics is infinitely deep and linked to all parts of mathematics.

— John Stillwell

Turn in:

(1) **Quick Proofs:** (None need be longer than one or two sentences)
   (a) Prove that for all integers $n > 1$ the inequality $2^n \leq \binom{2n}{n}$ holds.
   (b) Prove that for all positive integers n, the number $\binom{2n}{n}$ is even.
   (c) Prove that for all integers $n > k \geq 2$ the inequality $k^n \leq \binom{kn}{n}$ holds.

(2) Prove, by a combinatorial argument, that for all $n > 1$ the number $\binom{3n}{n,n,n}$ is divisible by 6.

(3) Find (and prove) a closed formula for $S(n, 2)$ if $n \geq 2$.

(4) Let a_n denote the number of compositions of n into parts that are larger than 1. Give a formula for a_n using a_{n-1} and a_{n-2}.

(5) We want to select as many subsets of $\{1, 2, 3, \ldots, n\}$ as possible without selecting two subsets where one subset contains the other. Prove that we can select at least $2^n/n$ subsets that do this.