Math 315 - Fall 2017
Homework 2
Due September 21, 2017

It is difficult to find a definition of combinatorics that is both concise and complete, unless we are satisfied with the statement "Combinatorics is what combinatorialists do"

— W.T. Tutte

Turn in:

(1) (Exercise 23, Chapter 2) Prove that for all positive integers n,
$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots n)^2$$

(2) (Exercise 31, Chapter 2) Let $a_1 = 1$ and let $a_{n+1} = 3 \cdot a_n + 4$ for $n \geq 1$. Prove that for all positive integers n the inequality $a_n \leq 3^n$ holds.

(3) (Exercise 42, Chapter 3) A host invites n couples to a party. She wants to ask a subset of the $2n$ guests to give a speech, but she does not want to ask *both* members of any couple to give speeches. In how many ways can she proceed? (Hint: your answer shouldn't include a summation sign or a ...)

(4) Prove the following identity:
$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$
Hint: Use a combinatorial proof. Let S be a set with three distinguished elements a, b, and c and count certain k-subsets of S.