Math 315 - Fall 2017
Homework 1
Due September 11, 2017

There is no problem in all mathematics that cannot be solved by direct counting.

— Ernst Mach

Turn in:

(1) Show that if $n+1$ distinct integers are chosen from the set $\{1, 2, \ldots, 2n\}$, then there are always two whose greatest common divisor is 1.

(2) (Exercise 24, Chapter 1) Find all 4-tuples (a, b, c, d) of distinct positive integers so that $a < b < c < d$ and
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1$$
Hint: Look at the solution to Exercise 2 in Chapter 1.

(3) Let S be a set of 17 points inside a cube of side length 1. Prove that there exists a sphere of radius 1/2 which encloses at least three of the points.

(4) For each n describe a sequence of n^2 numbers which does not contain a monotone increasing sequence of length $n+1$, nor a monotone decreasing sequence of length $n+1$. This shows that the Erdős-Szekeres Theorem is tight.