

Mid-Term Review

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Mission 4 Problem 4: Find the last two digits of $4^{3^{210}}$

Given that we are looking for the last two digits we will be using $(\text{mod}100)$

$$4^{3^{210}} (\text{mod}100)$$

$$3^{210} (\text{mod}\varphi100)$$

$$\varphi100=40$$

$$3^{210} (\text{mod}40) = 3$$

$$3^1 (\text{mod}40)=9 \quad 4^1 (\text{mod}100)=4$$

$$3^2 (\text{mod}40) = 9 \quad 4^2 (\text{mod}100)=16$$

$$3^4 (\text{mod}40) = 27 \quad 4^4 (\text{mod}100)=54$$

$$3^8 (\text{mod}40) = 1 \quad 4^8 (\text{mod}100)=36$$

$$3^{16} (\text{mod}40) = 1$$

$$3^{32} (\text{mod}40) = 1$$

$$3^{64} (\text{mod}40) = 1$$

$$3^{128} (\text{mod}40) = 1$$

$$210 = 128+64+16+2 \quad (9*1*1*1) \text{mod}40=9$$

$$4^9 (\text{mod}100)=4*36(\text{mod}100)=44$$

$$4^{3^{210}} (\text{mod}100)=44$$

Mission 4 Problem 3: Write down all of the 8 elements of field of 8 using the irreducible polynomial $x^3 + x + 1$

Multiply each element by $x^2 + 1$

| | |
|---------------|---------------|
| * | $x^2 + 1$ |
| 0 | 0 |
| 1 | $x^2 + 1$ |
| x | x |
| x+1 | $x^2 + x + 1$ |
| x^2 | x |
| $x^2 + 1$ | $x^2 + x + 1$ |
| $x^2 + x$ | x+1 |
| $x^2 + x + 1$ | $x^2 + x$ |

Missiong 5 Problem 4: Use Euclid's algorithm to find the inverse of $f(x) = x^2$ in the field F_8 with irreducible polynomial $x^3 + x + 1$

$$\begin{aligned} &\gcd(x^3 + x + 1, x^2) \\ x^3 + x + 1 &= x^2(x) + (x + 1) \\ x^2 &= (x+1)(x+1) + 1 \\ 1 &= x^2 - (x + 1)(x + 1) \\ x + 1 &= (x^3 + x + 1) - x(x^2) \\ 1 &= x^2 - (x + 1)((x^3 + x + 1) + x(x^2)) \\ &\quad (x^2 + x)(x^2) \\ 1 &= (x^2 + x + 1)(x^2) \pmod{x^3 + x + 1} \\ (x^2)^{-1} &= x^2 + x + 1 \end{aligned}$$

DES

The worksheet provided helps us encrypt 12-bit

-It is usually more than 12 bits though

So how would we encrypt 1000-bits?

-Encrypt 12-bits at a time.

Encrypting Plaintext longer than blocksize

First Idea: Break plaintext into chunks of size of a block, encrypt each block separately (*Mode of Operation*)