# MATH 314-Class Notes 

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Scribe: Kirill Vorobyev
Summary: Covered mini-mission 6, involving simple DES, with classmates comparing answers/strategies.
Went over SageMath SDES code demo and discussed, in depth, how the functions operate. Started discussion on differential cryptanalysis.

## Notes:

SageMath Code Demo Explanation:

- Start with list of lists containing $S_{1}$ and $S_{2}$
- Bin(S) converts numbers written with 1's and 0's into equivalent binary representation
- Expander (L) expands a string from 6 to 8 bits for SDES
- $\operatorname{XOR}(\mathrm{L}, \mathrm{M})$ performs binary xor addition on two string $\mathbf{L}, \mathbf{M}$ provided they are the same length
- roundkey (K,i) returns 8 bit key for round $i$ from master 9 bit key
- bind2int(L) converts binary list to integer
- split(M) splits list into two equal halves
- $\operatorname{SDES}\left(\mathrm{R}, K_{i}\right)$ performs function $f$
- SDESround (M, $K_{r}$ ) performs 1 round of SDES using roundkey $K_{r}$
- $\operatorname{SDES}(\mathrm{M}, \mathrm{K}, \mathrm{r})$ performs SDES on $\mathbf{M}$ with the key $\mathbf{K}$ using $\mathbf{r}$ rounds


## Differential Cryptanalysis

- One method to attack Feistel System Ciphers
- SDES is a Feistel System Cipher
- Recall what a Feistel System is

How differential cryptanalysis works on 3 rounds of SDES:

- Choose Plaintext
- Split into $L_{0}, R_{0}$
- Encrypt 3 times/rounds using SDES
- Start with known inputs $L_{0}$ and $R_{0}$
- $L_{0} \quad R_{0}$

1. $L_{1}=R_{0} \quad R_{1}=L_{0} \oplus f\left(R_{0}, K_{1}\right)$
2. $L_{2}=L_{0} \oplus f\left(R_{0}, K_{1}\right) \quad R_{2}=\ldots$
3. $L_{3}=R_{2} \quad R_{3}=L_{2} \oplus f\left(R_{2}, K_{3}\right)$

- End with known outputs $L_{3}$ and $R_{3}$
- $R_{3}=L_{0} \oplus f\left(R_{0}, K_{1}\right) \oplus f\left(R_{2}, K_{3}\right)$


## We want to find $\mathbf{K}$

Now choose a new plaintext $L_{0}^{*}, R_{0}^{*}$ where $R_{0}=R_{0}^{*}$ but $L_{0} \neq L_{0}^{*}$
Feed through SDES as well and get out:

- $R_{3}^{*}=L_{0}^{*} \oplus f\left(R_{0}^{*}, K_{1}\right) \oplus f\left(R_{2}^{*}, K_{3}\right)$
$R_{3}=L_{0} \oplus f\left(R_{0}, K_{1}\right) \oplus f\left(R_{2}, K_{3}\right) \leftarrow$ move down for reference
$R_{3} \oplus R_{3}^{*}=L_{0}^{*} \oplus L_{0} \oplus f\left(R_{2}^{*}, K_{3}\right) \oplus f\left(R_{2}, K_{3}\right)$
$\left(R_{3} \oplus R_{3}^{*}\right) \oplus\left(L_{0}^{*} \oplus L_{0}\right)=f\left(L_{3}^{*}, K_{3}\right) \oplus f\left(L_{3}, K_{3}\right)$
Note: $R_{2}^{*}=L_{3}^{*}$ and $R_{2}=L_{3}$
At this point we know $L_{0}, L_{0}^{*}, R_{3}, R_{3}^{*}, L_{3}, L_{3}^{*}$ and $f$. The only thing we do not know is $K_{3}$
$f\left(L_{3}, K_{3}\right): E\left(L_{3}\right) \oplus K_{3}$
This is input 1.
We must take the first 4 bits of the above and run them through S-box 1 to get our output $f\left(L_{3}^{*}, K_{3}\right): E\left(L_{3}^{*}\right) \oplus K_{3}$
This is input $1^{*}$.
We must take the first 4 bits of the above and run them through S-box 1 to get our output*
Note: Solving for input 1 is the same as solving for the first 4 bits of $K_{3}$
We don't know Input1 or Input1*
but if we figure out what they are we can get the first 4 bits of $K_{3}$
Note that Input1 $\oplus$ Input1* is the first 4 bits of
$\left(E\left(L_{3}\right) \oplus K_{3}\right) \oplus\left(E\left(L_{3}^{*}\right) \oplus\left(K_{3}\right)\right)=E\left(L_{3}\right)+E\left(L_{3}^{*}\right)$
We know Input1 $\oplus$ Input1*
Ex.
We know $L_{3}=101110$
We know $L_{3}^{*}=000010$
$E\left(L_{3}\right)=10111110$
$E\left(L_{3}^{*}\right)=00000010$
First 4 bits of $E\left(L_{3}\right) \oplus E\left(L_{3}^{*}\right)$ are 1011
This is Input1 $\oplus$ Input1*
We can use the fact that Input1 $\oplus$ Input1* $=1101$ to reduce the possible inputs into S-boxes to 16 possibilities

