(1) Find all first and second partial derivatives.

a.) $f(x, y) = x^2 \ln(x^2 + y^2)$

b.) $g(u, v) = \dfrac{u + 2v}{u^2 + v^2}$

(2) Find an equation of the tangent plane to the surface $z = e^x \cos y$ at the point $(0, 0, 1)$.

(3) Find the equation of the tangent plane to the level surface $7 = f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $(3, 2, 6)$.

(4) Show that the following limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y^6}{x^4 + y^{12}}.$$

(5) Let C be the curve with parametrization $\mathbf{r}(t) = t \sin t\, \mathbf{i} + t \cos t\, \mathbf{j}$. Find the equation of the tangent line to the curve C at $t = \frac{\pi}{4}$.

(6) Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation for S, but you know that the curves

$$r_1(t) = \langle 2 + 3t,\ 1 - t^2,\ 3 - 4t + t^2 \rangle, \quad \text{and}$$
$$r_2(u) = \langle 1 + u^2,\ 2u^3 - 1,\ 2u + 1 \rangle$$

both lie on S and pass through P. Find an equation of the tangent plane $T_P S$.

(7) Find the directional derivative of $f(x, y) = x^2 e^{-y}$ at the point $(-2, 0)$, in the direction of the point $(2, -3)$.

(8) *a.)* Find the gradient of f; *b.)* Evaluate the gradient at the point P; *c.)* Find the rate of change of f at the point P in the direction of the vector $\mathbf{u}$.

$$f(x, y) = \sin(2x + 3y)$$
$$P = (-6, 4)$$
$$\mathbf{u} = \frac{1}{2}(\sqrt{3}\mathbf{i} - \mathbf{j})$$

(9) Find the maximum rate of change of the function $f(x, y) = y\, e^{xy}$ at the point $(0, 2)$, and give the direction that it occurs.

(10) (a) Find the unique critical point of the function

$$f(x, y) = x^2 + 3xy + 2y^2 - 8x - 11y + 30.$$

(b) Is this critical point a minimum, maximum, or saddle?

(11) Find the absolute maximum and the absolute minimum of

$$f(x, y) = (x - 1)(y - 2)$$

in the closed triangle $0 \le x$, $0 \le y$, $x + y \le 7$ bounded by the x-axis, the y-axis, and the line $x + y = 7$.

(12) Find parametric equations for

(a) The plane through $(1, 3, 4)$ and orthogonal to $\mathbf{n} = \langle 2, 1, -1 \rangle$.

(b) The sphere centered at the origin and having radius 5.

(c) The sphere centered at the point $(2, -1, 3)$ and with radius 5.

(d) The cone $x^2 + y^2 = z^2$. $0 \le z \le 2$,

(13) Evaluate $\displaystyle\int_{x=0}^{1} \int_{y=2x}^{1} \int_{z=x^3+y}^{x^2+2y} y\, dz\, dy\, dx.$

(14) Let $\mathbf{F}(x,y) = x^2y\mathbf{i} + 2xy^2\mathbf{j}$. Compute

$$W = \int_C \mathbf{F} \cdot d\mathbf{r}$$

Where C is the line from $(0,0)$ to $(2,4)$ along the curve $y = x^2$.

(15) Evaluate the integral $\displaystyle\int\int_R x\,dx\,dy$ where R is the triangle with vertices $(1,2)$, $(3,3)$, $(4,5)$.

(16) Let $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j} = (x^2+y^2)\mathbf{i} + 2xy\mathbf{j}$ and Let γ be the curve which follows the parabola $y = x^2$ from $(0,0)$ to $(2,4)$, and the line segments from $(2,4)$ to $(0,4)$, and from $(0,4)$ to $(0,0)$. Use Green's theorem to evaluate $\int_\gamma \mathbf{F} \cdot \mathbf{dr}$.

(17) The cardioid is the curve with polar equation

$$r = 1 - \cos\theta, \qquad 0 \leq \theta \leq 2\pi.$$

and is given parametrically by $x(t) = \cos(t) - \cos^2(t)$, $y(t) = \sin(t) - \cos(t)\sin(t)$, $0 \leq t \leq 2\pi$.

Use Green's theorem to find the area of the region inside the cardioid by evaluating the integral

$$\oint_C x\,dy.$$

(18) Let S be the quarter of the disk of radius 1 in the yz-plane centered at the origin for which $y \geq 0$ and $z \geq 0$. Consider the vector field $\mathbf{F}(x,y,z) = \langle y, z, x\rangle$. Orient S so that its normal vector points in the direction of the positive x-axis.

(a) Give the boundary C of S the orientation induced by the right-hand rule. With this orientation, compute the line integral

$$\oint_C \mathbf{F} \cdot d\mathbf{s}.$$

(b) Compute the curl of the vector field, $\nabla \times \mathbf{F}$.

(c) Verify Stokes' Theorem by computing

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}.$$

(19) Let $W(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ and let $\mathbf{F} = \nabla W$ be the gradient of W.

(a) Calculate $\mathbf{F}$ and the divergence of $\mathbf{F}$, $\nabla \cdot \nabla W$.

(b) Use the divergence theorem to calculate the outward flux

$$\iint_\Sigma \mathbf{F} \cdot dS$$

through the surface σ which is the boundary of the solid S bounded by the xy-plane and by the hemispheres

$$z = \sqrt{4 - x^2 - y^2} \quad \text{and} \quad z = \sqrt{9 - x^2 - y^2}.$$