Math 275 - Spring 2016
Practice Midterm 2 Questions

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(1) Find the tangent plane to $f(x, y) = x^2 + 3xy - y^2$ at $(x, y) = (2, 3)$. $z = 13(x - 1)$

(2) Find an equation for the tangent plane to the surface given by $xy + yz^2 + z = 0$ at the point $(2, 1, 1)$. $(x - 2) + 3(y - 1) + 3(z - 1) = 0$

(3) You are standing above the point $(x, y) = (1, 3)$ on the surface $z = 20 - (2x^2 + y^2)$.
  (a) In which direction should you walk to ascend fastest? $\langle -\frac{4}{\sqrt{52}}, -\frac{6}{\sqrt{52}} \rangle$
  (b) If you start to move in this direction, what is the slope of your path when you first start to move? $\sqrt{52}$

(4) Consider the function $f(x, y) = x^3 + y^2 - 3x - 2y$.
  (a) Find and classify all critical points for the function.
  $(-1, 1)$: Saddle Point. $(1, 1)$: Minimum.
  (b) Find the absolute maximum and minimum of $f(x, y)$ in the rectangle with vertices $(0, 0)$, $(0, 1)$, $(1, 1)$, $(1, 0)$.
  The maximum is 0 at (0, 0) and the minimum is 3 at (1, 1).

(5) Let T be the triangle in the xy-plane with vertices $(0, 0), (1, 2)$, and $(2, 2)$. Find the area of the surface $z = 7 + \sqrt{8}x + y^2$ above T. $\frac{49}{12}$

(6) Change the order of integration to evaluate the given integral:
  (a) $\int_0^1 \int_{\sqrt{x}}^1 \frac{1}{\sqrt{1+y^3}}\, dy\, dx$ $\frac{2}{3}(\sqrt{2}-1)$
  (b) $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\sin^2 y}{y}\, dy\, dx$ $\frac{\pi}{4}$

(7) Let S be the solid in the first octant of 3-space bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 1$ and the coordinate planes xz and yz. Find and evaluate a triple integral in spherical coordinates that gives the volume of S. $\frac{(2-\sqrt{2})\pi}{12}$

(8) Find the volume of the solid bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = 2x + y$. $\frac{9}{20}$

(9) Let W be the three dimensional object that is bounded below by the cone $z = \sqrt{\frac{x^2+y^2}{3}}$ and bounded above by the sphere $x^2 + y^2 + z^2 = 9$. Suppose the density of W is distributed by

$$\delta(x, y, z) = e^{(x^2+y^2+z^2)^{3/2}}.$$

Find the mass of W. $\frac{\pi}{3}(e^{27} - 1)$

(10) Find the area of the surface with parametrization:

$$\mathbf{r}(u, v) = \langle \frac{1}{\sqrt{2}}u^2, uv, \frac{1}{\sqrt{2}}v^2 \rangle, \quad 0 \le u \le 1, 0 \le v \le 2.$$

$\frac{10\sqrt{2}}{3}$