**Math 275 - Spring 2016**
**Practice Midterm 2 Questions**

(1) Find the tangent plane to $f(x,y) = x^2 + 3xy - y^2$ at $(x,y) = (2,3)$.

(2) Find an equation for the tangent plane to the surface given by $xy + yz^2 + z = 0$ at the point $(2,1,1)$.

(3) You are standing above the point $(x,y) = (1,3)$ on the surface $z = 20 - (2x^2 + y^2)$.
  (a) In which direction should you walk to ascend fastest?
  (b) If you start to move in this direction, what is the slope of your path when you first start to move?

(4) Consider the function $f(x,y) = x^3 + y^2 - 3x - 2y$.
  (a) Find and classify all critical points for the function.
  (b) Find the absolute maximum and minimum of $f(x,y)$ in the rectangle with vertices $(0,0)$, $(0,1)$, $(1,1)$, $(1,0)$.

(5) Let $T$ be the triangle in the $xy$-plane with vertices $(0,0), (1,2)$, and $(2,2)$. Find the area of the surface $z = 7 + \sqrt{8}x + y^2$ above $T$.

(6) Change the order of integration to evaluate the given integral:
  (a) $\int_0^1 \int_{\sqrt{x}}^1 \frac{1}{\sqrt{1+y^3}}\, dy\, dx$
  (b) $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\sin^2 y}{y}\, dy\, dx$

(7) Let $S$ be the solid in the first octant of 3-space bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 1$ and the coordinate planes $xz$ and $yz$. Find and evaluate a triple integral in spherical coordinates that gives the volume of $S$.

(8) Find the volume of the solid bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = 2x + y$.

(9) Let $W$ be the three dimensional object that is bounded below by the cone $z = \sqrt{\frac{x^2+y^2}{3}}$ and bounded above by the sphere $x^2 + y^2 + z^2 = 9$. Suppose the density of $W$ is distributed by
$$\delta(x,y,z) = e^{(x^2+y^2+z^2)^{3/2}}.$$
  Find the mass of $W$.

(10) Find the area of the surface with parametrization:
$$\mathbf{r}(u,v) = \langle \frac{1}{\sqrt{2}}u^2, uv, \frac{1}{\sqrt{2}}v^2 \rangle, \quad 0 \le u \le 1, 0 \le v \le 2.$$