Math 275 - Spring 2016
Practice Midterm Questions

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(1) Identify the surface whose equation is given in cylindrical coordinates.
  (a) $z = 4 - r^2$. Downward facing elliptic paraboloid.
  (b) $2r^2 + z^2 = 1$. Ellipsoid.

(2) Identify the surface whose equation is given in spherical coordinates.
  (a) $\rho = \sin\theta \sin\phi$. Ellipsoid
  (b) $\rho^2(\sin^2\phi \sin^2\theta + \cos^2\phi) = 9$. Cylinder centered around x-axis.

(3) Find the angle between the given vectors, find the projection of $\mathbf{a}$ in the direction of $\mathbf{b}$, and find a unit vector orthogonal to both of them.
  (a) $\mathbf{a} = \langle 1, -1, 3\rangle$, $\mathbf{b} = \langle 1, 2, -1\rangle$ $\theta = \cos^{-1}\left(-2\sqrt{\frac{2}{33}}\right)$, $\langle -\frac{2}{3}, -\frac{4}{3}, \frac{2}{3}\rangle$, $\left\langle -\frac{1}{\sqrt{2}}, \frac{2\sqrt{2}}{5}, \frac{3}{5\sqrt{2}}\right\rangle$
  (b) $\mathbf{a} = \langle 3, -3, 2\rangle$, $\mathbf{b} = \langle 1, -1, 5\rangle$ $\theta = \cos^{-1}\left(\frac{8\sqrt{\frac{2}{33}}}{3}\right)$, $\langle \frac{16}{27}, -\frac{16}{27}, \frac{80}{27}\rangle$, $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\rangle$.

(4) Find the area of the triangle with vertices $A(1, 1, 1)$, $B(1, -1, 2)$, $C(0, 2, 3)$. $\frac{\sqrt{30}}{2}$

(5) Determine whether the lines L_1 and L_2 are parallel, skew or intersecting. If they intersect find the point of intersection.
  (a) L_1: $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-1}{-3}$. L_2: $\frac{x-3}{1} = \frac{y+4}{3} = \frac{z-2}{-7}$.Intersect: $(4, -1, -5)$
  (b) L_1: $x = 3 + 2t$, $y = 4 - t$, $z = 1 + 3t$. L_2: $x = 1 + 4s$, $y = 3 - 2s$, $z = 4 + 5s$. Skew

(6) Find vector and linear equations for each plane.
  (a) The plane containing the point $(1, 3, 2)$ with normal vector $\langle 2, -1, 5\rangle = 0$. $(\langle x, y, z\rangle - \langle 1, 3, 2\rangle) \cdot \langle 2, -1, 5\rangle = 0$, $2x - y + 5z - 9 = 0$.
  (b) The plane containing the points $(2, 0, 3)$, $(1, 1, 0)$ and $(3, 2, 1)$. $(\langle x, y, z\rangle - \langle 2, 0, 3\rangle) \cdot \langle 4, -5, -3\rangle = 0$, $4x - 5y + -3z + 1 = 0$.
  (c) The plane containing the point $(1, 2, 1)$ and perpendicular to the planes $x + y = 2$ and $2x + y - z = 1$. $(\langle x, y, z\rangle - \langle 1, 2, 1\rangle) \cdot \langle -1, 1, -1\rangle = 0$, $-x + y - z = 0$.

(7) Find the distance between the point $(2, 0, 1)$ and the plane $2x - y + 2z = 4$. $\frac{2}{3}$

(8) Find the equation of the tangent line to the curve $\mathbf{r}(t) = \langle t, 4\cos(\pi t), t^2 - 1\rangle$ at the point $(2, 4, 3)$. $\mathbf{l}(t) = \langle 1, 0, 4\rangle\, t + \langle 2, 4, 3\rangle$

(9) Find the arclength of $\mathbf{r}(t) = \left\langle e^t, e^{-t}, t\sqrt{2}\right\rangle$ for $0 \le t \le 2$. $e^2 - e^{-2}$

(10) **(Corrected)** Find the point of intersection and the angle between the curves $\mathbf{f}(s) = \langle s, s^2, s^2 + s\rangle$ and $\mathbf{g}(t) = \langle t - 3, 1, t^2 - 4\rangle$. Intersect: (-1,1,0), $\theta = \cos^{-1}\left(-\sqrt{\frac{3}{34}}\right)$.

(11) For each function draw the level curves and describe the graph of the function.
  (a) $f(x, y) = 2x^2 - y$. Nested Parabolas, (rotated) Parabolic cylinder
  (b) $f(x, y) = (x - 2)^2 + (y + 1)^2$. Concentric circles around (2,-1), Upward facing Paraboloid

(12) Find the limit if it exists, or show that the limit does not exist.
  (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$. DNE
  (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^3\cos(y)}{2x^3 + y^5}$. DNE
  (c) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{((x-1)^2 + (y-2)^2)^2}{(x-1)^2 + (y-2)^2}$ 0

(13) Classify each surface.

(a) $x^2 - y^2 + z^2 - 4x - 2z = 0$. One Sheeted Hyperboloid

(b) $4x^2 + y^2 + z^2 - 24x - 8y + 4z + 55 = 0$. Ellipsoid