1. Let $\mathbf{r}(t)$ be a vector function with values in $\mathbf{R}^3$: $\quad \mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$.

   (a) Its derivative is $\frac{d\mathbf{r}}{dt} = \left(\frac{df}{dt}, \frac{dg}{dt}, \frac{dh}{dt}\right)$.

   (b) The magnitude of this vector is $||\frac{d\mathbf{r}}{dt}|| = \sqrt{\left(\frac{df}{dt}\right)^2 + \left(\frac{dg}{dt}\right)^2 + \left(\frac{dh}{dt}\right)^2}$.

   (c) The unit tangent vector is $\mathbf{T} = \frac{\frac{d\mathbf{r}}{dt}}{||\frac{d\mathbf{r}}{dt}||}$.

   (d) The vector equation of the line tangent to the graph of $\mathbf{r}(t)$ at the point $P = (x_0, y_0, z_0)$ corresponding to $t = t_0$ on the curve is $\mathbf{R}(t) = \mathbf{r}_0 + (t - t_0)\,\mathbf{v}_0$, where $\mathbf{r}_0 = \mathbf{r}(t_0)$ and $\mathbf{v}_0 = \frac{d\mathbf{r}}{dt}(t_0)$.

   (e) The arc length of the graph of $\mathbf{r}(t)$ between t_1 and t_2 is $L = \int_{t_1}^{t_2} ||\frac{d\mathbf{r}}{dt}||\,dt$.

2. Let Σ be a surface in $\mathbf{R}^3$: $z = f(x, y)$

   (a) The slope f_x of the surface in the x-direction at the point (x_0, y_0) is $f_x(x_0, y_0) = \frac{\partial f}{\partial x}(x_0, y_0)$.

   (b) The slope f_y of the surface in the y-direction at the point (x_0, y_0) is $f_y(x_0, y_0) = \frac{\partial f}{\partial y}(x_0, y_0)$.

   (c) The equation for the tangent plane to the surface at the point $P = (x_0, y_0, z_0)$ is $z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$.

   (d) The volume under the surface and over a region R in the xy-plane is $V = \iint_R f(x, y)\,dA$.

   (e) The area of the portion of the surface that is above a region R in the xy-plane is $S = \iint_\Sigma dS = \iint_R \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA$.

   (f) The mass of the lamina with the density $\delta(x, y, z)$ that is the portion of the surface that is above a region R in the xy-plane is $M = \iint_\Sigma \delta(x, y, z)\,dS = \iint_R \delta(x, y, z)\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA$.

3. The local linear approximation of the function $z = f(x, y)$ at the point (x_0, y_0) is $L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$.

4. Let $f(x, y, z)$ be a function of three variables

   (a) The gradient of f is $\nabla f = (f_x, f_y, f_z)$.

   (b) f increases most rapidly in the direction of its gradient, and the rate of change of f in this direction is equal to $||\nabla f||$.

   (c) If f is smooth then its critical points satisfy $f_x = f_y = f_z = 0$.

5. Let R be a region in the xy-plane bounded by the curves $y = g(x)\,,\ y = h(x)\,,\ x = a\,,\ x = b$, and $g \leq h$ for $a \leq x \leq b$. Then the double integral over the region is
$\iint_R f(x,y)\,dA = \int_a^b \left[\int_{g(x)}^{h(x)} f(x,y)dy\right] dx\,.$

6. Let R be a region in the xy-plane bounded by the curves (in polar coordinates)
$r = r_1(\theta)\,,\ r = r_2(\theta)\,,\ \theta = \alpha\,,\ \theta = \beta$ and $r_1 \leq r_2$ for $\alpha \leq \theta \leq \beta$. Then the double integral over the region is
$\iint_R f(r,\theta)\,dA = \iint_R f(r,\theta)\,rdr\,d\theta = \int_\alpha^\beta \left[\int_{r_1(\theta)}^{r_2(\theta)} f(r,\theta)rdr\right] d\theta\,.$

7. Let R be a plain lamina with density $\delta(x,y)$.
   (a) Its mass is equal to $M = \iint_R \delta(x,y)\,dA\,.$
   (b) The x-coordinate of its center of mass is equal to $\overline{x} = \frac{1}{M}\iint_R x\,\delta(x,y)\,dA\,.$
   (c) The y-coordinate of its center of mass is equal to $\overline{y} = \frac{1}{M}\iint_R y\,\delta(x,y)\,dA\,.$

8. Let G be a simple solid whose projection onto the xy-plane is a region R. G is bounded by a surface $z = g(x,y)$ from below and by a surface $z = h(x,y)$ from above.
   (a) The triple integral over the solid is $\iiint_G f(x,y,z)\,dV = \iint_R \left[\int_{g(x,y)}^{h(x,y)} f(x,y,z)\,dz\right] dA\,.$
   (b) The volume of the solid is $V = \iiint_G dV\,.$

9. Let G be a solid enclosed between the two surfaces (in spherical coordinates)
$\rho = g(\theta\,,\phi)\,,\ \rho = h(\theta\,,\phi)$, for $0 \leq \theta_1 \leq \theta \leq \theta_2 \leq 2\pi$, $0 \leq \phi_1 \leq \phi \leq \phi_2 \leq \pi$, .
   (a) The triple integral over the solid is
   $\iiint_G f(\rho,\theta,\phi)\,dV = \int_{\theta_1}^{\theta_2}\left(\int_{\phi_1}^{\phi_2}\left[\int_{g(\theta\,,\phi)}^{h(\theta\,,\phi)} f(\rho,\theta,\phi)\,\rho^2 d\rho\right]\sin\phi\,d\phi\right) d\theta\,.$
   (b) The volume of the solid is $V = \iiint_G dV = \int_{\theta_1}^{\theta_2}\left(\int_{\phi_1}^{\phi_2}\left[\int_{g(\theta\,,\phi)}^{h(\theta\,,\phi)} \rho^2 d\rho\right]\sin\phi\,d\phi\right) d\theta\,.$
   (c) The mass of the solid with the density $\delta(\rho,\theta,\phi)$ is $M = \iiint_G \delta(\rho,\theta,\phi)\,dV\,.$

10. The area of the surface that extends upward from the curve $x = x(t)\,,\ y = y(t)\,,\ a \leq t \leq b$ in the xy-plane to the surface $z = f(x,y)$ is given by the following line integral
$A = \int_C z\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt\,.$

11. Consider a line integral $\int_C P(x,y)\,dx + Q(x,y)\,dy$, and let $A = (x_0,y_0)$ and $B = (x_1,y_1)$ be the endpoints of the curve C.
   (a) If the vector field $\langle P(x,y), Q(x,y)\rangle$ is conservative, then there is a potential function $f(x,y)$ satisfying $\frac{\partial f}{\partial x} = P(x,y)\,,\ \frac{\partial f}{\partial y} = Q(x,y)\,,$

(b) and the Fundamental Theorem of Line Integrals says that
$\int_C P(x,y)\,dx + Q(x,y)\,dy = \int_A^B \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy = f(x_1,y_1) - f(x_0,y_0)\,.$

12. Let a closed curve C be oriented counterclockwise, and be the boundary of a simply connected region R in the xy-plane. By Green's Theorem we have

$$\int_C P(x,y)\,dx + Q(x,y)\,dy = \iint_R \left(\frac{\partial Q(x,y)}{\partial x} - \frac{\partial P(x,y)}{\partial y}\right) dA$$

13. Let $\mathbf{F}(x,y,z) = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ be a vector field.

(a) If Σ is the surface $z = f(x,y)$, oriented by upward unit normals $\mathbf{n}$, lying above the region R in the xy-plane then
flux $= \iint_\Sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R \left(-P\frac{\partial f}{\partial x} - Q\frac{\partial f}{\partial y} + R\right) dA\,.$

(b) According to the Divergence Theorem the flux of $\mathbf{F}$ across a closed surface Σ with outward orientation is
flux $= \iint_\Sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_V \operatorname{div}\mathbf{F}\,dV\,, \quad \operatorname{div}\mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\,.$

(c) If Σ is an oriented smooth surface that is bounded by a simple, closed, smooth boundary curve C with positive orientation then, according to Stokes' Theorem
$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_\Sigma (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dS\,,\ \operatorname{curl}\mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{k}\,.$