Math 314 - Fall 2016 **Name:**_________________________

Worksheet 2 November 1st, 2016

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(1) Let $f(x) = (x+1)^5 - 5x + 2$. Find the intervals of increase or decrease, the local maximum and minimum values, intervals of concavity and the inflection points.

Compute: $f'(x) = 5(x+1)^4 - 5$, $f''(x) = 20(x+1)^3$.

To find critical points set $f'(x) = 0$: $5 = 5(x+1)^4$ so $(x+1)^4 = 1$, thus the critical points are $x = -2, 0$. We can check that $f'(x) > 0$ for $x < -2$, $f'(x) < 0$ for $-2 < x < 0$ and $f'(x) > 0$ for $x > 0$. Thus f is increasing on $(-\infty, -2) \cup (0, \infty)$ and decreasing on $(-2, 0)$. The first derivative test shows that $x = -2$ is a local max and $x = 0$ is a local min.

Using the second derivative we find that $f''(x) = 0$ when $x = -1$. Since $f''(x) < 0$ when $x < -1$ and $f''(x) > 0$ when $x > -1$, we have that $x = -1$ is an inflection point, and is concave down on $(-\infty, -1)$ and concave up on $(-1, \infty)$.

(2) Let $g(x) = x\sqrt{6-x}$. Find the intervals of increase or decrease, the local maximum and minimum values, intervals of concavity and the inflection points.

Note that the domain of $g(x)$ is $x \leq 6$.

Compute:

$$g'(x) = -\frac{x}{2\sqrt{6-x}} + \sqrt{6-x},$$

$$g''(x) = -\frac{\sqrt{6-x} + \frac{x}{2\sqrt{6-x}}}{2(6-x)} - \frac{1}{2\sqrt{6-x}}$$
$$= -\frac{1}{\sqrt{6-x}} - \frac{x}{4(6-x)^{3/2}}$$

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To find critical points set $g'(x) = 0$: $\sqrt{6-x} = \frac{x}{2\sqrt{6-x}}$ so $2(6-x) = x$, thus the only critical point is $x = 4$. We can check that $g'(x) > 0$ for $x < 4$, $g'(x) < 0$ for $4 < x < 6$ and $f'(x) > 0$ for $x > 0$. Thus f is increasing on $(-\infty, 4)$ and decreasing on $(4, 6)$.

The first derivative test shows that $x = 4$ is a local maximum.

Setting $g''(x) = 0$ we get $-\frac{1}{\sqrt{6-x}} = \frac{x}{4(6-x)^{3/2}}$. Multiplying both sides by $4(6-x)^{3/2}$ we get $-4(6-x) = x$ when $x = 8$. Since this is outside the domain the function has no inflection points and since $g''(x) < 0$ it is concave down on $(\infty, 6)$.

(3) Compute the following limits.

(a) $\lim_{x \to 0} \frac{\cos(x) - 1}{3x}$

This has the indeterminant form $\frac{0}{0}$. Using L'Hospital's rule this becomes

$$\lim_{x \to 0} \frac{\sin x}{3} = 0.$$

(b) $\lim_{x \to 0} \frac{\ln(x+1)}{e^x - 1}$

This has the indeterminant form $\frac{0}{0}$. Using L'Hospital's rule this becomes

$$\lim_{x \to 0} \frac{\frac{1}{x+1}}{e^x} = 1.$$