Math 273 - Fall 2015
Practice Midterm 3

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(1) Find the (exact) maximum and minimum values of $f(x) = x - 2\sin x$ on the interval $[0, 2\pi]$. Minimum: $\frac{\pi}{3} - \sqrt{3}$ at $x = \frac{\pi}{3}$. Maximum: $\frac{5\pi}{3} + \sqrt{3}$ at $x = \frac{5\pi}{3}$.

(2) Let $f(x) = x^3 - 4x$.

(a) Find the intervals on which f is increasing or decreasing. Increasing: $(-\infty, -\frac{2}{\sqrt{3}}) \cup (\frac{2}{\sqrt{3}}, \infty)$, Decreasing: $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$.

(b) Find the local maxima and minima of f. Local Maximum: $\left(-\frac{2}{\sqrt{3}}, \frac{16}{3\sqrt{3}}\right)$, Local Minimum: $\left(\frac{2}{\sqrt{3}}, -\frac{16}{3\sqrt{3}}\right)$.

(c) Find the intervals on which f is concave up or concave down. Concave down: $(-\infty, 0)$, Concave up: $(0, \infty)$.

(d) Find the inflection points of f. (0,0)

(e) Sketch the curve.

(3) Find the limits of the following functions.

(a) $\lim_{x\to 0^+} \frac{\sqrt{x}}{\ln x}$. 0

(b) $\lim_{x\to 0} \frac{\sin(x) - x}{x^3}$. $-\frac{1}{6}$

(c) $\lim_{x\to 0} \frac{e^x - 1}{\sin(x) - 1}$. 0

(4) Find $f(x)$ given the information below.

(a) $f'(x) = x - \sqrt{x} + 1/x$. $f(x) = \frac{x^2}{2} - \frac{2x^{3/2}}{3} + \ln|x| + C$

(b) $f'(x) = \sin(x) + \cos(x)$. $f(x) = -\cos(x) + \sin(x) + C$

(c) $f''(x) = 4e^x + 1$, $f'(0) = 1$, $f(0) = 2$. $f(x) = 4e^x + \frac{x^2}{2} - 3x - 2$

(5) Find the rectangle of largest area that can be inscribed in a semicircle of radius r. Area: r^2

(6) Find the point on the line $y = 2x + 3$ that is closest to the origin. $\left(-\frac{6}{5}, \frac{3}{5}\right)$

(7) A stone is dropped off of a cliff. It accelerates downward due to gravity at 32 ft/sec and hits the ground travelling 120 ft/sec. How high was the cliff? 225 feet

(8) Show that the equation $f(x) = 2x + \cos x = 0$ has exactly one real root. Since $f(-\frac{\pi}{2}) = -\pi$ and $f(\frac{\pi}{2}) = \pi$ it must have at least one root by the Intermediate value theorem. However since $f'(x) = 2 - \sin(x) > 0$ for all x, $f(x)$ is increasing for all values of x, and therefore cannot have more than one root.