

Math 273 - Fall 2015
Practice Midterm

(1) Find the derivative of the following functions.

(a) $f(\theta) = \frac{\sec \theta}{1+\tan \theta} \cdot \frac{\sec \theta \tan \theta}{1+\tan \theta} - \frac{\sec^3 \theta}{(1+\tan \theta)^2}$

(b) $y(x) = \sqrt{e^x + 1} \cdot \frac{e^x}{2\sqrt{e^x+1}}$

(c) $h(t) = \ln t + \frac{1}{\arcsin t} \cdot \frac{1}{t} + \frac{1}{\arcsin(t)^2 \sqrt{1-t^2}}$

(d) $g(x) = \frac{x(x+2)^2(x-1)^8}{\sqrt{x^3(x+3)e^x}} \cdot \frac{x(x+2)^2(x-1)^8}{\sqrt{x^3(x+3)e^x}} \left(\frac{1}{x} + \frac{2}{x+2} + \frac{8}{x-1} - \frac{1}{2} \left(\frac{3}{x} + \frac{1}{x+3} + e^x \right) \right)$

(e) $r(x) = x^{1/x} \cdot x^{1/x} \left(\frac{1}{x^2} - \frac{\ln x}{x^2} \right)$

(f) $h(x) = \sinh(\sqrt{x}) + \cosh(1/x) \cdot \frac{\cosh(\sqrt{x})}{2\sqrt{x}} - \frac{\sinh(1/x)}{x^2}$

(2) Let $f(x) = \sin(2x)$. What is $f'(x)$? What is $f''(x)$? What is $f'''(x)$? What is $f^{(4)}(x)$? What is $f^{(42)}(x)$? Give a formula for $f^{(n)}(x)$ valid for all positive integers n . (Here $f^{(n)}(x)$ means the n -th derivative of $f(x)$.) $f'(x) = 2 \cos(2x)$, $f''(x) = -4 \sin(2x)$, $f'''(x) = -8 \cos(2x)$, $f^{(4)}(x) = 16 \sin(2x)$. $f^{(42)}(x) = -2^{42} \sin(2x)$.

$$f^{(n)}(x) = \begin{cases} (-1)^{n/2} 2^n \sin 2x & n \text{ even} \\ (-1)^{(n-1)/2} 2^n \cos 2x & n \text{ odd} \end{cases}$$

(3) What is the derivative of $\ln(x)$? Prove your answer using implicit differentiation. $1/x$
(Show work)

(4) At what point on the curve $y = [\ln(x + 4)]^2$ is the tangent line horizontal? -3

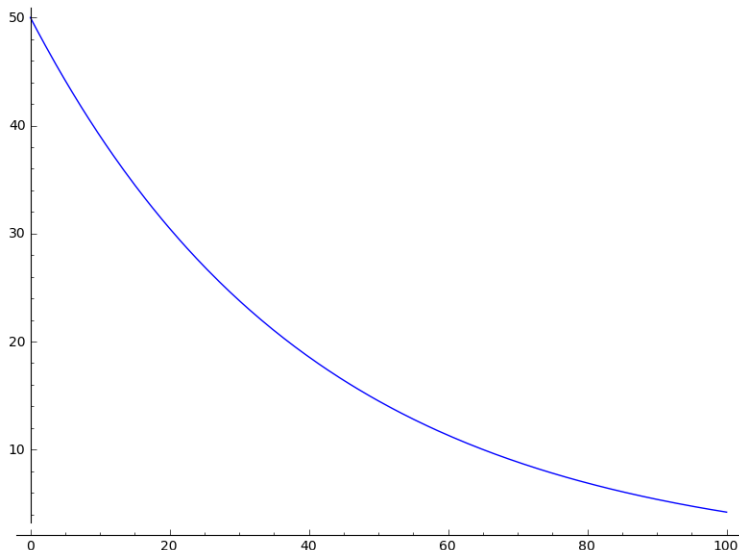
(5) Strontium-90 has a half life of 28 days.

(a) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after t days.

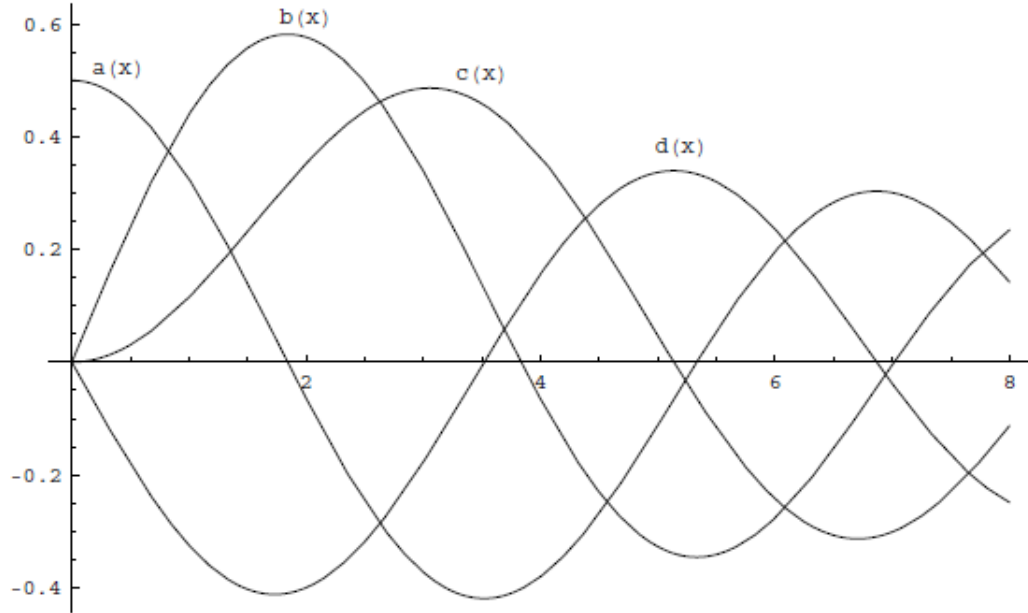
$$m(t) = 50e^{\frac{\ln(1/2)}{28}t}$$

(b) Find the mass remaining after 40 days. $50e^{\frac{\ln(1/2)}{28}40}$

(c) sketch a graph of the mass function.



- (6) Find the equation to the tangent line to $y^2 + \ln(y - 1) = x^2(3 - x)$ at the point $(2,2)$.
 $y = -\frac{1}{4}x + 2.5$
- (7) A man walks along a straight path at a speed of 3 ft/s. A searchlight is located on the ground 15 feet from the path and is kept focused on the man. At what rate (in radians/sec) is the searchlight rotating when the man is 30 feet from the searchlight?
 $\frac{d\theta}{dt} = \frac{\sqrt{675}}{300}$
- (8) Below are the graphs of a function $f(x)$, together with $f'(x)$, $f''(x)$ and an unrelated function $g(x)$. Identify each.



$f(x) = b(x)$, $f'(x) = a(x)$, $f''(x) = d(x)$, $c(x)$ is unrelated.