Math 273 - Fall 2015
Practice Midterm

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(1) Find the derivative of the following functions.

   (a) $f(\theta) = \frac{\sec\theta}{1+\tan\theta}$. $\frac{\sec\theta\tan\theta}{1+\tan\theta} - \frac{\sec^3\theta}{(1+\tan\theta)^2}$

   (b) $y(x) = \sqrt{e^x + 1}$. $\frac{e^x}{2\sqrt{e^x+1}}$

   (c) $h(t) = \ln t + \frac{1}{\arcsin t}$. $\frac{1}{t} + \frac{1}{\arcsin(t)^2\sqrt{1-t^2}}$

   (d) $g(x) = \frac{x(x+2)^2(x-1)^8}{\sqrt{x^3(x+3)e^x}}$. $\frac{x(x+2)^2(x-1)^8}{\sqrt{x^3(x+3)e^x}}\left(\frac{1}{x} + \frac{2}{x+2} + \frac{8}{x-1} - \frac{1}{2}\left(\frac{3}{x} + \frac{1}{x+3} + e^x\right)\right)$

   (e) $r(x) = x^{1/x}$. $x^{1/x}\left(\frac{1}{x^2} - \frac{\ln x}{x^2}\right)$

   (f) $h(x) = \sinh(\sqrt{x}) + \cosh(1/x)$. $\frac{\cosh(\sqrt{x})}{2\sqrt{x}} - \frac{\sinh(1/x)}{x^2}$

(2) Let $f(x) = \sin(2x)$. What is $f'(x)$? What is $f''(x)$? What is $f'''(x)$? What is $f^{(4)}(x)$? What is $f^{(42)}(x)$? Give a formula for $f^{(n)}(x)$ valid for all positive integers n. (Here $f^{(n)}(x)$ means the n-th derivative of $f(x)$.) $f'(x) = 2\cos(2x)$, $f''(x) = -4\sin(2x)$, $f'''(x) = -8\cos(2x)$, $f^{(4)}(x) = 16\sin(2x)$. $f^{(42)}(x) = -2^{42}\sin(2x)$.

$$f^{(n)}(x) = \begin{cases} (-1)^{n/2}2^n \sin 2x & n \text{ even} \\ (-1)^{(n-1)/2}2^n \cos 2x & n \text{ odd} \end{cases}$$

(3) What is the derivative of $\ln(x)$? Prove your answer using implicit differentiation. $1/x$ (Show work)

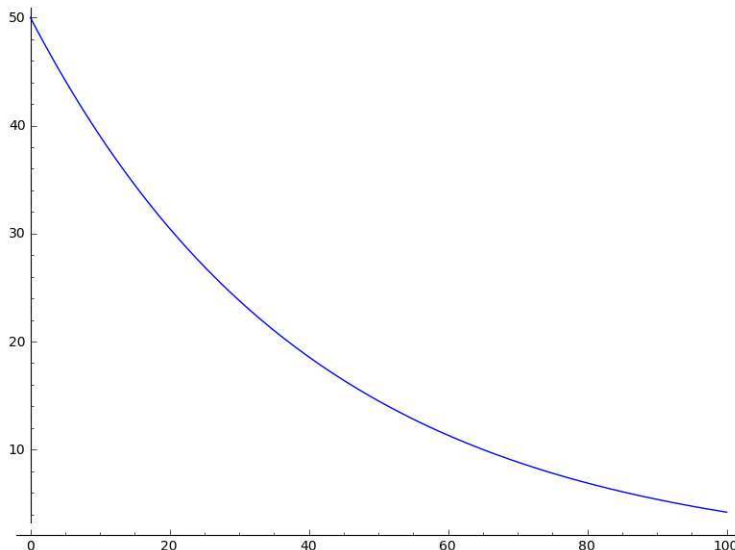(4) At what point on the curve $y = [\ln(x+4)]^2$ is the tangent line horizontal? -3

(5) Strontium-90 has a half life of 28 days.

   (a) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after t days.
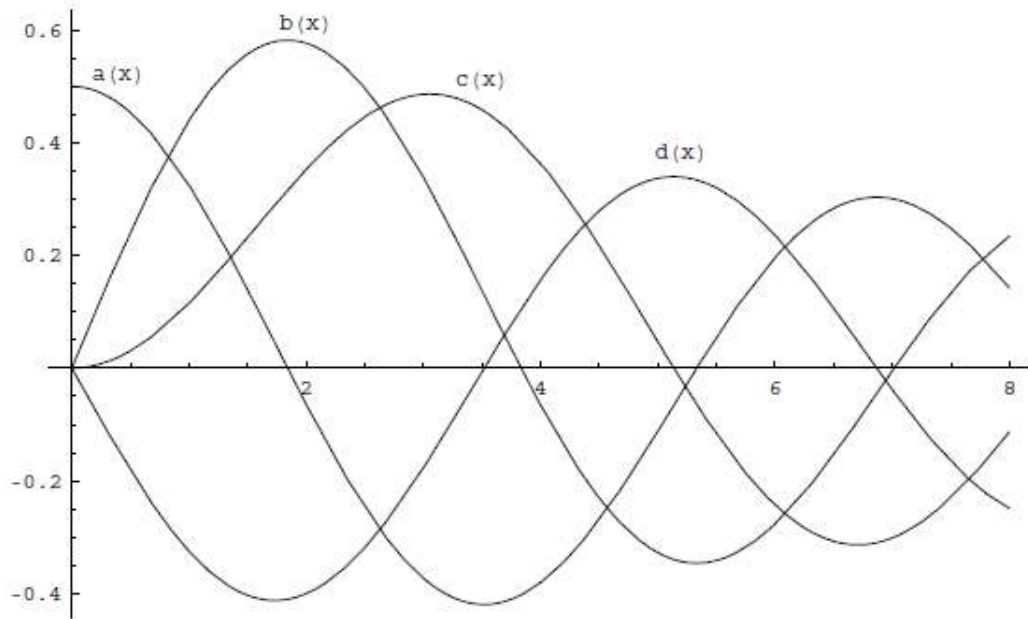
$$m(t) = 50e^{\frac{\ln(1/2)}{28}t}$$

   (b) Find the mass remaining after 40 days. $50e^{\frac{\ln(1/2)}{28}40}$

   (c) sketch a graph of the mass function.

(6) Find the equation to the tangent line to $y^2 + \ln(y - 1) = x^2(3 - x)$ at the point (2,2).
$y = -\frac{1}{4}x + 2.5$

(7) A man walks along a straight path at a speed of 3 ft/s. A searchlight is located on the ground 15 feet from the path and is kept focused on the man. At what rate (in radians/sec) is the searchlight rotating when the man is 30 feet from the searchlight?
$\frac{d\theta}{dt} = \frac{\sqrt{675}}{300}$

(8) Below are the graphs of a function $f(x)$, together with $f'(x)$, $f''(x)$ and an unrelated function $g(x)$. Identify each.

$f(x) = b(x), f'(x) = a(x), f''(x) = d(x)$, c(x) is unrelated.