

Name Key (Answers only, not work)

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Evaluate the limit  $\lim_{h \rightarrow 0} \frac{\sqrt{h^2 + 9} - 3}{h^2} = \frac{1}{6}$

2. Find all of the horizontal and vertical asymptotes of the graph of the function

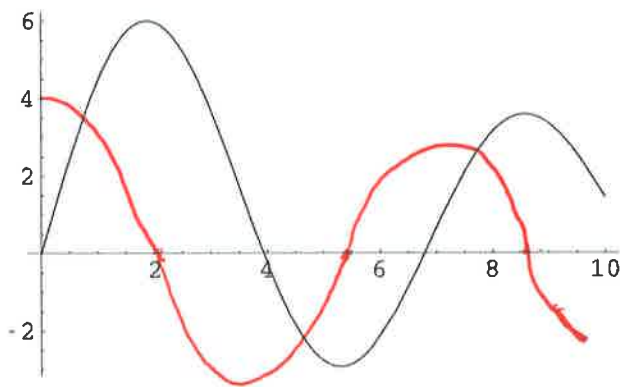
$$f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}$$

vertical @  $x = \frac{5}{3}$   
horizontal @  $\frac{\sqrt{2}}{3}$

3. Find the equation of the tangent line to the curve  $y = x^2 + 2x$  that passes through the point  $(-3, 3)$ .

$y = -4x - 9$

4. Here is a graph of a function  $f(x)$ . Sketch  $f'(x)$ .



5. Find the derivative:

a.  $y(x) = x^2 + 3x - 5$

$2x + 3$

b.  $f(x) = \frac{1}{x} + \sqrt{x}$

$-\frac{1}{x^2} + \frac{1}{2\sqrt{x}}$

c.  $g(t) = e^t - \frac{10}{\sqrt{t}}$

$e^t + \frac{5}{t^{3/2}}$  (Not on our midterm)

§2 Comprehension

6. What is the informal definition of limit? What is the precise definition of limit? Use the precise definition to prove that  $\lim_{x \rightarrow 2} (3x - 1) = 5$ .

See book for definitions, For  $\epsilon > 0$  let  $\delta = \frac{\epsilon}{3}$

Then if  $|x - 2| < \delta = \frac{\epsilon}{3}$   
then  $|3x - 6| < \epsilon$   
so  $|(3x - 1) - 5| < \epsilon$ .

7. Prove that  $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$ . Use this fact to prove that if

By squeeze thm,  
 $-|x| < x \sin(\frac{1}{x}) < x$

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$f'(0)$   
 $= \lim_{a \rightarrow 0} \frac{f(a) - f(0)}{a - 0}$   
 $= \lim_{a \rightarrow 0} \frac{a^2 \sin \frac{1}{a} - 0}{a - 0} = \lim_{a \rightarrow 0} a \sin \frac{1}{a} = 0$

then  $f$  is differentiable at  $x = 0$ . Find  $f'(0)$ .

8. What does it mean for a function to be differentiable at the point  $x = a$ ? Prove that if  $f$  is differentiable at  $x = a$ , then it is continuous at  $x = a$ .

See book for definition.

9. What is the Intermediate Value Theorem? Use it to prove that there is a root of the equation

$$4x^3 - 6x^2 + 3x - 2 = 0$$

between 1 and 2.

### §3 Application

10. If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters after  $t$  seconds is given by  $h(t) = 58t - 0.83t^2$ .

- What is the arrow's average velocity over the time interval  $[1, 2]$ ? The interval  $[1, 1.1]$ ? The interval  $[1, 1.01]$ ?
- What is the arrow's instantaneous velocity after one second?

This fn is continuous b/c it is a polynomial.

$f(1) = -1$   $f(2) = 12$   
so IVT  $\Rightarrow$  There exists  $1 < c < 2$   $f(c) = 0$

a) 55.51, 56.257, 56.3317

b) 56.34