Math 273 Fall 2015

Calculus I

Midterm Exam 2

Wednesday October 21 6:00-8:00 PM

Your name (please print): ______________________________

Instructions: This is a closed book exam. You may not refer to the textbook, your notes or any other material, including a calculator. Answers do not need to be simplified, however you should **provide enough work to explain how you arrived at your answer.**

The academic integrity policy requires that you neither give nor receive any aid on this exam.

For grader use only:

Problem	Points	Score
1	30	
2	6	
3	8	
4	10	
5	7	
6	8	
7	10	
Total	79	

1. Compute the derivatives of the following functions.

   a. $f(x) = (x^2 + \sqrt{x})^5$
   $5(x^2 + \sqrt{x})^4 \left(2x + \dfrac{1}{2\sqrt{x}}\right)$

   b. $g(x) = \dfrac{2\cos(5x)}{\ln(x)} + e^5$
   $\frac{\ln(x)(-10\sin(5x)) - 2\cos(5x)\frac{1}{x}}{\ln(x)^2} = \frac{-10x\ln(x)\sin(5x) - 2\cos(5x)}{x\ln(x)^2}$

   c. $h(x) = \tan(x)\sqrt{x^2+1}$
   $\tan(x)\frac{1}{2\sqrt{x^2+1}}(2x) + \sec^2(x)\sqrt{x^2+1}$

d. $y = \dfrac{x^{10}(x-3)^2\sqrt{x-1}}{2(x+3)(x-2)^{3/4}}$

$$\begin{aligned}\ln y &= \ln\left(\frac{x^{10}(x-3)^2\sqrt{x-1}}{2(x+3)(x-2)^{3/4}}\right)\\ &= 10\ln(x) + 2\ln(x-3) + \frac{1}{2}\ln(x-1) - \ln 2 - \ln(x+3) - \frac{3}{4}\ln(x-2)\end{aligned}$$

$$\frac{y'}{y} = \frac{10}{x} + \frac{2}{x-3} + \frac{1}{2(x-1)} - \frac{1}{x+3} - \frac{3}{4(x-2)}$$

$$y' = \left(\frac{10}{x} + \frac{2}{x-3} + \frac{1}{2(x-1)} - \frac{1}{x+3} - \frac{3}{4(x-2)}\right)\left(\frac{x^{10}(x-3)^2\sqrt{x-1}}{2(x+3)(x-2)^{3/4}}\right)$$

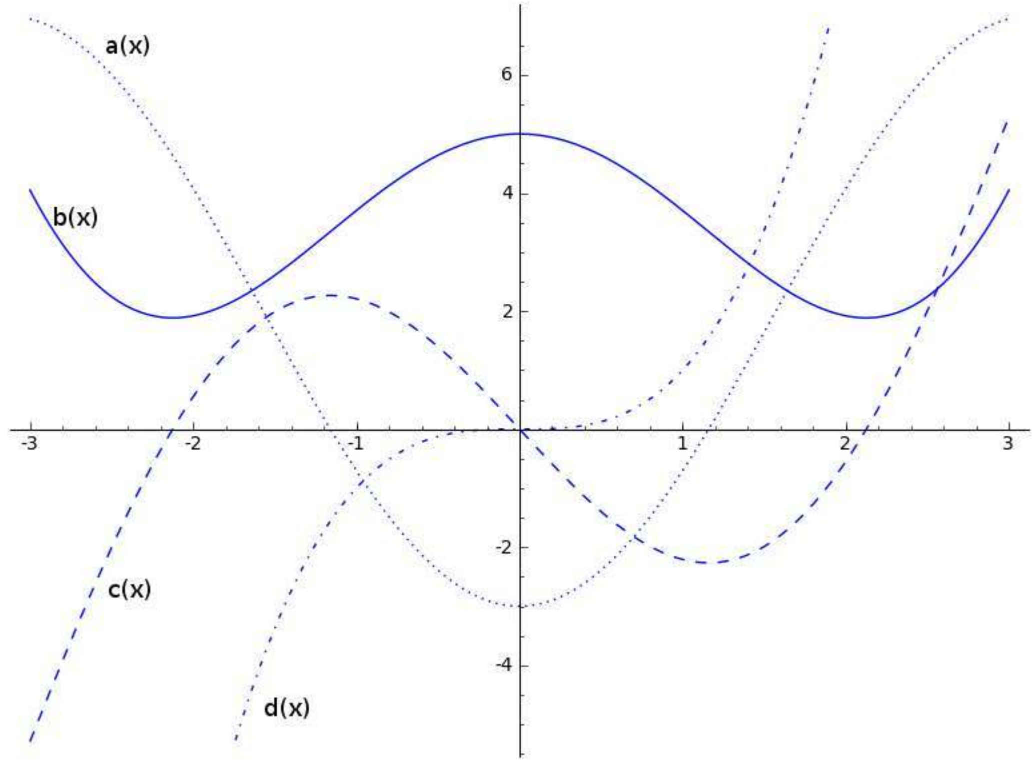
e. $\sin\left(\tan^{-1}(x)\right)$

$$\frac{\cos(\tan^{-1}(x))}{1+x^2}$$

f. $\cosh(\ln(\tan(x))$

$$\sinh(\ln(\tan(x)))\left(\frac{1}{\tan(x)}\right)\sec^2(x)$$

2. Below are the graphs of a function, $f(x)$, together with $f'(x)$, $f''(x)$, and an unrelated function $g(x)$. Identify each.



$f(x) =$ b(x) $\qquad$ $f'(x) =$ c(x) $\qquad$ $f''(x) =$ d(x) $\qquad$ $g(x) =$ a(x)

3. Find the values of x where the equation $y = 4x^2 + \frac{1}{x}$ has a horizontal tangent line. $y' = 8x - \frac{1}{x^2}$. Setting this equal to zero, we get $0 = 8x - \frac{1}{x^2}$, so $8x = \frac{1}{x^2}$ or $x^3 = \frac{1}{8}$. Thus $x = \frac{1}{2}$.

4. Today (October 21, 2015) is the date that Marty travels to in the movie "Back the Future II." According to the movie, it is necessary to drive the time machine at 88 MPH in order to make it travel back through time.

   The time machine is driving down a road which lies $\frac{\sqrt{5}}{4}$ miles away from a clocktower at its closest point. If it reaches 88 mph (traveling down the road) at the instant when it is $\frac{3}{4}$ miles away from the clocktower, how fast is the time machine moving away from the clocktower at that instant? If a is the position of the car along the road (from the point closest to the clocktower) and c is the distance of the car from the clocktower, then $a^2 + \frac{5}{16} = c^2$. Thus, when $c = \frac{3}{4}$, $c^2 = \frac{9}{16}$, so $a = \frac{4}{16} = \frac{1}{4}$.
   Differentiating $a^2 + \frac{5}{16} = c^2$ with respect to t, we have $2aa' = 2cc'$. Plugging in $a = \frac{1}{4}$, $c = \frac{3}{4}$, $a' = 88$ and solving for $c = \frac{\frac{1}{4}88}{\frac{3}{4}} = \frac{88}{3}$.

5. Find the equation of the tangent line to the curve $e^{xy-1} = x^2 - x - 2y$ through the point (2,1/2). Differentiating with respect to x:

$$e^{xy-1}(xy' + y) = 2x - 1 - 2y'.$$

   Solving for y':

$$y'(2 + xe^{xy-1}) = 2x - 1 - ye^{xy-1}$$

   so

$$y' = \frac{2x - 1 - ye^{xy-1}}{(2 + xe^{xy-1})}.$$

   Plugging in $x = 2, y = 1/2$:

$$y' = \frac{4 - 1 - \frac{1}{2}}{4} = \frac{5}{8}.$$

   Thus our tangent line is $y = \frac{5}{8}x + b$, where b is chosen such that the line goes through the point $(2, 1/2)$. So $b = -\frac{3}{4}$.

$$y = \frac{5}{8}x - \frac{3}{4}$$

6. Suppose that electrons are flowing through a wire to charge a battery. If the amount of charge (measured in coulombs) accumulated after t seconds is given by $C(t) = \frac{100t}{1+2t}$.

   (a) If the battery were allowed to charge forever, how many coulombs of charge would accumulate in the battery?

$$\lim_{t\to\infty} \frac{100t}{1+2t} = \lim_{t\to\infty} \frac{100}{1/t+2} = 50 \text{ Coulombs}$$

   (b) What is the current (in amperes) flowing into the battery after 1 second? (Recall that 1 ampere = 1 coulomb/second.)

$$C'(t) = \frac{(1+2t)100 - 100t(2)}{(1+2t)^2}$$

So

$$C'(1) = \frac{300-200}{9} = \frac{100}{9} \text{ Amperes.}$$

   (c) Sketch a graph of the amount of charge in the battery after t seconds.

7. The population of the US is currently 320 Million, and was 280 million in the year 2000. Assume that the rate of change of the population, $P(t)$ of the US (where t is years since the year 2000) is given by $P'(t) = kP(t)$, for some value of k.

   (a) What is the value of k? (You do not need to simplify your answer, you can leave it as a function involving the natural log.) We know $P(t) = Ce^{kt}$ for some C and k. Since $P(0) = C = 280,000,000$, and $P(15) = 280000000e^{15k} = 320000000$, we can solve for k,

   $$e^{15k} = \frac{32}{28} = \frac{8}{7}$$

   So

   $$k = \frac{\ln(7/8)}{15}.$$

   (b) What will the population of the US be in the year 2100?

   $$P(100) = 280000000e^{100\ln(7/8)/15} = 280000000(7/8)^{20/3}.$$