# Math 273

Final Examination

December 13, 2005

Name______________________________

*All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.*

## §1 Calculation

1. Evaluate:

   a. $\lim_{h\to 0} \frac{\sqrt{h^2+9}-3}{h^2}$. $\frac{1}{6}$

   b. $\lim_{x\to 0} \frac{\cos x - 1}{x^2}$. $-\frac{1}{2}$

   c. $\lim_{x\to 0} \frac{e^x - 1}{\sin x - 1}$. $-\infty$

   d. $\lim_{x\to 0+} x^{\sin x}$. $1$

2. Differentiate

   a. $g(s) = \sqrt{s} - e^{2s} + \tan(s) + \ln(s)$ $\quad g'(s) = \frac{1}{2\sqrt{s}} - 2e^{2s} + \sec^2(s) + \frac{1}{s}$

   b. $q(y) = \frac{(y^3 - 3y^2 + y)}{y^2+1}$ $\quad q'(y) = \frac{(y^2+1)(3y^2-6y+1) - (y^3-3y^2+y)(2y)}{(y^2+1)^2}$

   c. $y(x) = \sqrt[4]{\frac{2+x^2\cos(x)}{x^4+x^2+5}}$ $\quad y'(x) = \frac{1}{4}\left(\frac{2+x^2\cos(x)}{x^4+x^2+5}\right)\left(\frac{(x^4+x^2+5)(2x\cos(x)+x^2(-\sin(x)))}{(x^4+x^2+5)^2} - \frac{(2+x^2\cos(x))(4x^3+2x)}{(x^4+x^2+5)^2}\right)$

   d. $f(x) = x^{\sin x}$.

3. The graph of the curve $2(x^2+y^2)^2 = 25(x^2-y^2)$ is called a lemniscate. Find $dy/dx$. Find the equation of the tangent line to the curve that passes through $(1,3)$. $\quad \frac{dy}{dx} = -\frac{8x(x^2+y^2) - 50x}{50y + 8y(x^2+y^2)}$

   $\boxed{y = -\frac{1}{13} + \frac{40}{13}}$

4. Find the exact maximum and minimum values of the given functions on the indicated intervals

   a. $f(t) = 7t(\ln(t) - 9)$ on $[1,8]$. Min: $-63$ Max: $56(\ln(8)-9)$

   b. $x(y) = 2y - \frac{15}{\sqrt[3]{y}}$ on $[1,6]$ Min: $-13$ Max: $12 - \frac{15}{\sqrt[3]{6}}$

5. Consider the function $f(x) = x\ln x - 2x$.

   a. Find the domain of $f$. $(0,\infty)$

   b. Find the asymptotes of $f$. $x=0$

   c. Find the intervals on which $f$ is increasing or descreasing. Decreasing: $(0,e)$ Increasing $(e,\infty)$

   d. Find the local maxima and minima of $f$. Local min: $(e,-e)$

   e. Find the intervals on which $f$ is concave up or concave down. Concave up: $(0,\infty)$

   f. Find the inflection points of $f$. None

   g. Sketch the curve.