MATH 267 Fall 2026 - Class Notes

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Summary: Today, we went over the best approach to prove a theorem. We practiced proving theorems using different methods such as, direct proofs, contrapositives, and proof by contradiction.

Notes: Include detailed notes from the lecture or class activities. Format your notes nicely using latex such as

- If we have a theorem that takes the form of an implication. Proofs should always start with "Let P . . ." or "Suppose P . . .". We need to write out the assumptions we are making.
- At the end of a proof, we need to conclude with something like "therefore . . . Q".

Definitions: Every definition stated in class should appear here, word for word as we stated it. This is the part your classmates will actually come back to.

Definition 1. *An integer x is **even** if $x = 2n$ for some integer n.*

Theorems and Proofs:

Theorem 1. *If $a \mid b$ and $b \mid c$, then $a \mid c$.*

Proof. Suppose that a, b, c are integers, and $a \mid b$ and $b \mid c$. The definition of divides says that since $a \mid b$, there exists an integer q so that $b = aq$ and since $b \mid c$, there exists another integer p such that $c = pb$. We can substitute b for aq. That gives us $c = p(aq) = (pq)a = ra$ if $r = pq$. Since $c = pb$ and $b = aq$, we have $c = p(aq)$ by substitution. So $c = (pq)a$ where $pq = r$ is an an integer. Therefore, $c = ra$. □

Theorem 2. *If x and y are real numbers with $x + y \geq 2$ then $x \geq 1$ or $y \geq 1$.*

A direct proof did not work, so we will try a contrapositive. The contrapositive is $\neg Q \rightarrow \neg P$. It gives us If x and y are real numbers with $x < 1$ and $y < 1$, then $x + y < 2$.

Proof. Suppose that $\{x, y \in \mathbb{R}$ and $x < 1$ and $y < 1$. Then, $x + y < 1 = 2$. Therefore, $x + y < 2$. This proves the contrapositive of if $x + y \geq 2$, then $x \geq 1$ or $y \geq 1$. □

Questions raised in class: There are some theorems presented at the end of the class that we did not get to finish.

- The sum of even integer and an odd integer is odd.
- The a/b/c/d/x/y are integers with $a \mid b$ and $a \mid c$ then $a \mid (bx + cy)$.
- If a,b,c are integers and $a \nmid bc$ then $a \nmid b$.