

MATH 267 Fall 2026 - Class Notes

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Scribe: Marin Colella

Summary: We started with the warm-up and went over the answers. Then, we discussed what induction proofs look like and what we need to include in them. We then took a quiz. We ended class by working on a few problems to work with induction in groups.

Notes:

Warm-up discussion:

- The OEIS website contains catalogs of integer number sequences
- The difference between regular induction and strong induction: Regular: need a base case - $P(1)$, and an induction step - $P(n) \rightarrow P(n+1)$.
- Strong: need base case(s) - $P(n_0)$, along with $P(n_0+1)$, and as many more as we need for our formulas to work. We also need an induction step - $P(n_0) \wedge P(n_0+1) \wedge P(n) \rightarrow P(n+1)$. In strong induction, we assume our statement holds for every value up to and including n . Use all of those to prove it for $n+1$.

Example: Prove $a_n = 3a_{n-1} - 2a_{n-2}$ using strong induction. Proof: base cases

- $n = 1, a_1 = 2^1 - 1, 1 = 2 - 1$
- $n = 2, a_2 = 2^2 - 1, 3 = 4 - 1.$
- Both base cases hold

Induction step: Induction hypothesis: Suppose $a_k = 2^k - 1$ for all $1 \leq k \leq n$ (this formula holds for every index up to n). The goal is to show it holds for a_{n+1} . Now, $a_{n+1} = 3a_n - 2a_{n-1}$. Using our induction hypothesis for n and $n-1$. Thus, $a_{n+1} = 2(2^n) - 3 - (2^n) + 2 = (3-1)2^n - 1 = 2(2^n) - 1 = 2^{n+1} - 1$, which is the formula we wanted to prove.

- We then took our quiz and class break
- Example: Which numbers can we write as a sum using 4's and 5's? Can we write $n = 4k + 5l$? **See table 1.** Idea: use strong induction showing that if $n-4$ can be written using this formula, then n can. Since we look back at 4 cases in our proof, we need 4 base cases.

Proof. Base cases:

$$12 = 4(3) + 5(0)$$

$$13 = 4(2) + 5(1)$$

n	$4k + 5l$
1	X
2	X
3	X
4	$4(1)$
5	$5(1)$
6	X
7	X
8	$2(4)$
9	$1(4) + 1(5)$
10	$2(5)$

Table 1: Which n can be written as $4k + 5l$

$$14 = 4(1) + 5(2)$$

$$15 = 4(0) + 5(3)$$

Induction step: Suppose every integer $12 \leq m \leq n$ can be written as $m = 4(k) + 5(l)$ for nonnegative integers k and l , and $n \geq 16$. Then $n + 1 = (n - 3) + 4$ and $n - 3 \geq 12$ since $n \geq 16$. So, by our induction hypothesis, $n - 3 = 4k + 5l$ for some nonnegative integers k and l . Adding 4 to both sides, $n - 3 + 4 = 4k + 5l + 4$, so $(n + 1) = 4(k + 1) + 5l$. $\square$

Definitions:

Definition 1. *Postage stamp problems: determining which integer values can or can't be formed by adding together a given set of numbers.*

Definition 2. *A formula like $a_{n+1} = 3a_n - 2a_{n-1}$ where terms depend on previous terms is called a recursive formula.*

Definition 3. *A formula that doesn't depend on previous terms, like $a_n = 2^n - 1$, is called a closed formula.*

Theorems and Proofs:

Theorem 1. *Every number $n \geq 12$ can be written as $n = 4k + 5l$ for nonnegative integers k and l .*

Prove $a_n = 3a_{n-1} - 2a_{n-2}$ using strong induction.

Proof. Base cases:

- $n = 1, a_1 = 2^1 - 1, 1 = 2 - 1$
- $n = 2, a_2 = 2^2 - 1, 3 = 4 - 1.$
- Both base cases hold

Induction step: Induction hypothesis: Suppose $a_k = 2^k - 1$ for all $1 \leq k \leq n$ (this formula holds for every index up to n). The goal is to show it holds for a_{n+1} . Now, $a_{n+1} = 3a_n - 2a_{n-1}$. Using our induction hypothesis for n and $n - 1$. Thus, $a_{n+1} = 2(2^n) - 3 - (2^n) + 2 = (3 - 1)2^n - 1 = 2(2^n) - 1 = 2^{n+1} - 1$, which is the formula we wanted to prove. $\square$

Questions raised in class: No questions were raised in class that weren't settled.